THE

NATURAL ARITHMETIC.

SPECIALLY PREPARED

FOR ELEMENTARY SCHOOLS.

BY

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PREFACE.

To the Friends of Elementary Schools: —

The Natural Arithmetic has been prepared to meet a demand which has grown out of the experience of live, progressive teachers during the last ten years. Many educators have learned that a large part of the matter in common arithmetics is of no practical use to nine tenths of the children in our elementary schools; and that the fundamental principles involved in the use of numbers are *few;* but that they are enshrouded in many blind and useless rules. At least one half of the time usually spent in studying arithmetic is wasted in memorizing rules and processes, and in unraveling mysteries which yield no profit even when solved.

The study of arithmetic is required, not only for its practical value, but also because of its supposed disciplinary advantages. But while it is not evident that arithmetic has superior advantages in the way of discipline, it *is* evident that more than one half of the labor usually required to flounder through the uncomprehended matter of the common text-books is destitute of any valuable discipline or practical advantage. In this work, the design has been to eliminate everything not necessary to secure sound discipline and practical training. It is not claimed that this work is perfect or exhaustive; but that it is a step in the right direction and is sufficiently exhaustive to furnish all the arithmetical training and practical knowledge of numbers needed during the first five or six years of school training. It will furnish all the practical and theoretical knowledge of arithmetic needed to fit the pupil for any of the common employments of life and for more advanced arithmetical and mathematical training.

While this work furnishes all that any pupil needs to know of arithmetic, during the first six years of school, it also furnishes hints to the teachers as to the best methods of teaching it. Nearly all the first part of this book is to be studied by the teacher, and the principles taught orally by the use of the blackboard and slate. Many of the rules and suggestions are *new* and well proved.

Z. RICHARDS.

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CHAPTER IV.

CHAPTER V.

CHAPTER VI.

MULTIPLICATION AND DIVISION TABLES.

As it is essential that these Tables should be memorized at as early a period as possible, the following method of learning them is recommended: —

$2\times 1= 1\times 2= 2$ and $2\div 1= 2$ and $1\div 2=\frac{1}{2}$
$2\times 2= 4$ and $4\div 2= 2$
$2\times 3= 3\times 2= 6$ and $6\div 2= 3$ and $6\div 3=2$
or, one half of $6=3$ and one third of $6=2$, etc.
$2\times 4= 4\times 2= 8$ and $8\div 2= 4$ and $8\div 4=2$
$2\times 5= 5\times 2=10$ and $10\div 2= 5$ and $10\div 5=2$
$2\times 6= 6\times 2=12$ and $12\div 2= 6$ and $12\div 6=2$
$2\times 7= 7\times 2=14$ and $14\div 2= 7$ and $14\div 7=2$
$2\times 8= 8\times 2=16$ and $16\div 2= 8$ and $16\div 8=2$
$2\times 9= 9\times 2=18$ and $18\div 2= 9$ and $18\div 9=2$
$2\times 10=10\times 2=20$ and $20\div 2=10$ and $20\div 10=2$

$3\times 3= 9$ and $9\div 3= 3$ and $1\div 3=\frac{1}{3}$
$3\times 4= 4\times 3=12$ and $12\div 3= 4$ and $12\div 4=3$
$3\times 5= 5\times 3=15$ and $15\div 3= 5$ and $15\div 5=3$
$3\times 6= 6\times 3=18$ and $18\div 3= 6$ and $18\div 6=3$
$3\times 7= 7\times 3=21$ and $21\div 3= 7$ and $21\div 7=3$
$3\times 8= 8\times 3=24$ and $24\div 3= 8$ and $24\div 8=3$
$3\times 9= 9\times 3=27$ and $27\div 3= 9$ and $27\div 9=3$
$3\times 10=10\times 3=30$ and $30\div 3=10$ and $30\div 10=3$

$4\times 4=16$ and $16\div 4= 4$ and $1\div 4=\frac{1}{4}$
$4\times 5= 5\times 4=20$ and $20\div 4= 5$ and $20\div 5=4$
$4\times 6= 6\times 4=24$ and $24\div 4= 6$ and $24\div 6=4$
$4\times 7= 7\times 4=28$ and $28\div 4= 7$ and $28\div 7=4$
$4\times 8= 8\times 4=32$ and $32\div 4= 8$ and $32\div 8=4$
$4\times 9= 9\times 4=36$ and $36\div 4= 9$ and $36\div 9=4$
$4\times 10=10\times 4=40$ and $40\div 4=10$ and $40\div 10=4$

$5 \times 5 = 25$ and $25 \div 5 = 5$ and $1 \div 5 = \frac{1}{5}$
$5 \times 6 = 6 \times 5 = 30$ and $30 \div 5 = 6$ and $30 \div 6 = 5$
or, one fifth of $30 = 6$ and one sixth of $30 = 5$, etc.
$5 \times 7 = 7 \times 5 = 35$ and $35 \div 5 = 7$ and $35 \div 7 = 5$
$5 \times 8 = 8 \times 5 = 40$ and $40 \div 5 = 8$ and $40 \div 8 = 5$
$5 \times 9 = 9 \times 5 = 45$ and $45 \div 5 = 9$ and $45 \div 9 = 5$
$5 \times 10 = 10 \times 5 = 50$ and $50 \div 5 = 10$ and $50 \div 10 = 5$

$6 \times 6 = 36$ and $36 \div 6 = 6$ and $1 \div 6 = \frac{1}{6}$
$6 \times 7 = 7 \times 6 = 42$ and $42 \div 6 = 7$ and $42 \div 7 = 6$
or, one sixth of $42 = 7$ and one seventh of $42 = 6$, etc.
$6 \times 8 = 8 \times 6 = 48$ and $48 \div 6 = 8$ and $48 \div 8 = 6$
$6 \times 9 = 9 \times 6 = 54$ and $54 \div 6 = 9$ and $54 \div 9 = 6$
$6 \times 10 = 10 \times 6 = 60$ and $60 \div 6 = 10$ and $60 \div 10 = 6$

$7 \times 7 = 49$ and $49 \div 7 = 7$ and $1 \div 7 = \frac{1}{7}$
$7 \times 8 = 8 \times 7 = 56$ and $56 \div 7 = 8$ and $56 \div 8 = 7$
$7 \times 9 = 9 \times 7 = 63$ and $63 \div 7 = 9$ and $63 \div 9 = 7$
$7 \times 10 = 10 \times 7 = 70$ and $70 \div 7 = 10$ and $70 \div 10 = 7$

$8 \times 8 = 64$ and $64 \div 8 = 8$ and $1 \div 8 = \frac{1}{8}$
$8 \times 9 = 9 \times 8 = 72$ and $72 \div 8 = 9$ and $72 \div 9 = 8$
$8 \times 10 = 10 \times 8 = 80$ and $80 \div 8 = 10$ and $80 \div 10 = 8$

$9 \times 9 = 81$ and $81 \div 9 = 9$ and $1 \div 9 = \frac{1}{9}$
$9 \times 10 = 10 \times 9 = 90$ and $90 \div 9 = 10$ and $90 \div 10 = 9$

$10 \times 10 = 100$ and $100 \div 10 = 10$ and $1 \div 10 = \frac{1}{10}$

DIRECTIONS.—*First*, require the pupils to *read* the above tables in concert, and separately, until they can do it readily.

Second, require them to memorize the tables, so that they can recite them without mistake.

INTRODUCTION.

(To be studied carefully by the teacher.)

This work is specially designed to provide for the arithmetical instruction needed during the first five or six years of school life. It recognizes the necessity that children should learn how to read arithmetical characters and language understandingly and readily, and that they should be trained to think arithmetically.

All arithmetical characters are signs of ideas or things, just as words are, and should be taught upon the same plan.

The first step in arithmetic, then, is to teach the child to associate one, two, three, or more things, as balls, pebbles, fingers, blocks, etc., with the oral or printed word-sign first, and then with the numeral sign, so that the sign will suggest, at once, the number of things, and that the number of things seen will suggest the sign. We see that arithmetical language is both oral and written, like any other language. If a child, for the first time, sees an apple, he does not think of two or more apples, but of a single apple.

He should therefore learn the oral or written sign of it, thus, *one* apple, or *1* apple, as the first step in learning to count. Counting does not alone consist in saying *one*, *two*, *three*, etc., up to one hundred, but in knowing that these signs represent one, two, three, or more things, as balls, apples, etc. If, for instance, the child should count the fingers on one hand, by saying *one*, *two*, *three*, *four*, *five*, he should be taught that when he reaches the fifth finger it is not five, or five fingers, but that all the fingers make five fingers, that the last finger counted is not finger five, but the *fifth* finger. The meanings of all kinds of arithmetical terms should be so learned that the pupils can use them orally and understandingly.

If the term or sign for *five* is used alone, it always represents *five* units of some kind, as *five* ones, *five* tens, *five* hundredths, *five* shillings, *five* pounds, etc., and the same idea should be taught in regard to any other arithmetical term representing any denomination. These terms may represent the number of equal parts of things, as halves, tenths, fifths, etc.

Again, these signs change their names and their meaning; first, by a change of position; and, second, by increasing or diminishing their denominate value.

CLASSES OF NUMBERS.

There are only four classes or kinds of numbers which can be changed by increase or diminution.

1. There are *whole* numbers, or units of the same name.

2. There are *decimal* parts of whole numbers, as tenths of a unit, tenths of a tenth, etc., representing *any unit* divided into ten equal parts.

3. There are *common fractional* numbers, when any whole number is divided into any number of equal parts except tenths, such as halves, thirds, fourths, etc.

4. There are *denominate* numbers, or units of different names, used in the same expression, where a certain number of units of a lower value, required to make a unit of the next higher value, are expressed together, as four pounds, six shillings, and eight pence; or two hogsheads, five gallons, three quarts, one pint, and two gills; written, also, 2 hhd., 5 gal., 3 qts., 1 pt., 2 gi.

METHODS OF OPERATING WITH NUMBERS.

After learning how to *read* all the four kinds of numbers, there are, by universal consent, *four* methods of using them.

1. Any number may be increased by *adding* one or more numbers to it. The process is called addition.

2. Any number may be diminished by taking one or more numbers from it, or *out of* it. This process is called subtraction.

3. Any number may be increased by repeating it any number of times. This process is called multiplication.

4. Any number may be diminished, measured, or divided, by taking any smaller number from it a certain number of times, or by finding how many times a smaller number may be taken from, or is contained in, a larger. This process is called division.

From what has been said above, we see that each of the four classes of numbers is subject to four kinds of operations. It is, therefore, evident that when these *four kinds of numbers* and *four methods of operations* are clearly understood, the pupil will easily acquire sufficient knowledge of pure arithmetic for all the practical purposes of life.

FUNDAMENTAL PRINCIPLES.

1. The pupils should be taught to observe and keep in mind the name and meaning of every number used.

2. They should also learn and keep in mind the number of equal parts of any name or denomination required to make a unit of the next higher denomination.

3. If numbers to be compared are not of the same name and form they must be made so, and then they can be used as simple whole numbers.

The above statement of arithmetical principles is believed to be sufficient to prepare the way for a brief, clear, accurate, and sufficiently comprehensive understanding of all the principles of arithmetic which are needed in any business of life.

WHAT DOES ARITHMETIC REQUIRE?

1. A knowledge of arithmetic requires an ability to read and understand arithmetical language. The signs which represent numbers, whether the characters are single or combined, figures or letters, must be thoroughly learned. Thus the character 5, or the word five, or the letter V, seen or heard, should always convey the idea of *five things*, of some kind.

If 5 is combined with 6, thus 56; or L combined with VI, thus LVI, the language means fifty-six *things*. The 5 changes its value, or becomes ten times greater, by being placed first on the left side of the unit figure, and a hundred times greater by being placed third from the unit; thus increasing ten times at each removal to the left.

As soon as possible, the pupils should be taught to write the numbers which they can read. In reading numbers special care should be taken to make pupils understand the difference between cardinal and ordinal numbers. In giving the number of a page, chapter, or verse, it should not be read page four, chapter ten, etc., but page *fourth*, chapter *tenth*, or fourth page, etc.

This last direction may be considered, by some, as incorrect, or at least unimportant; but it is grammatically correct, and will prevent erroneous ideas in counting. The error referred to is very common with teachers, preachers, and professional men generally.

2. A knowledge of arithmetic implies also the ability to add, or combine, the values of all kinds of numbers, accurately and rapidly, so as to be able to give the amounts at once when the numbers are seen or heard. Generally, no part of arithmetical training is more superficial and imperfect than that of addition, though the work of addition is more frequently required than that of any other part of arithmetic.

3. This knowledge also implies the ability to find the difference between two or more numbers, or to diminish one number by taking one or more numbers from it; that is, by subtracting one number from another. The numbers must always be of the same kind, or made to be of the same kind, before they can be subtracted.

4. This knowledge also implies the ability to find and give the products of numbers, by repetition, as accurately and rapidly as by addition; or, in other words, to multiply any one of the four classes of numbers, first, by any one of the nine digits used as units, and, second, used as tens; third, used as hundreds, and so on, which is really only a short method of adding.

5. Again, this knowledge implies the ability to divide any of the four kinds of numbers, or to diminish any number by finding out how many times a smaller number is contained in it, which is really only a short method of subtraction.

Every arithmetical operation is included under one or another of the above-named requisites for an arithmetical education.

The principles involved in the foregoing discussion must be clearly understood by every teacher before he is properly qualified to teach arithmetic.

The following course of arithmetical training should be accompanied with experimental and practical examples.

CHAPTER I.

READING OF NUMBERS.

Section 1. READING OF WHOLE NUMBERS.

Numbers, like other things, are represented by written or printed characters. The characters are as follows: 1. or one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine; and 0, cipher.

Table of Representation.

●, 1 ball, or one ball.
●●, 2 balls, or two balls.
●●●, 3 balls, or three balls.
●●●●, 4 balls, or four balls.
●●●●●, 5 balls, or five balls.
●●●●●●, 6 balls, or six balls.
●●●●●●●, 7 balls, or seven balls.
●●●●●●●●, 8 balls, or eight balls.
●●●●●●●●●, 9 balls, or nine balls.
●●●●●●●●●●, 10 balls, or ten balls.

The nine digits can be so used as to represent any number of things. As before intimated, whole numbers, beginning with units, increase tenfold in value for each place to the left, and decrease tenfold in value for each place to the right. The first place is called the units' place; the second place on the left is tens; the third place is hundreds; the fourth place is thousands; the fifth place is tens of thousands; the sixth place is hundreds of thousands, and the seventh place is called millions. The period of millions has three places: millions, tens of millions, and hundreds of millions; and the next period of billions has also three places: billions, tens of billions, and hundreds of billions; and so on through trillions, etc.

The first period of three numbers to the right of units is tenths, hundredths, and thousandths; the second period is ten-thousandths, hundred-thousandths, and millionths, etc.

The following methods of reading whole numbers and decimals are recommended for use until the pupils can read all common whole numbers and decimals readily.

Billions	3,
Hundred-Millions ...	4
Ten-Millions........	9
Millions............	8,
Hundred-Thousands ..	7
Ten-Thousands	6
Thousands	5,
Hundreds	4
Tens	3
Units	**2.**
Tenths..............	3
Hundredths	4
Thousandths	5,
Ten-Thousandths.....	6
Hundred-Thousandths.	7
Millionths..........	8,
Ten-Millionths......	9
Hundred-Millionths..	4
Billionths	3

Another Method of Reading Whole Numbers.

Units take one place	as 2.	Two.
Tens take two places	„ 20.	Twenty.
Hundreds take three places	„ 400.	Four hundred.
Thousands take four places	„ 5,000.	Five thousand.
Ten-thousands take five places	„ 60,000.	Sixty thousand.
Hundred-thousands take six places . .	„ 700,000.	Seven hundred thousand.
Millions take seven places	„ 8,000,000.	Eight million.

If the significant figures of the last method are read in their order, beginning with the last, the number will be 8,765,422; or eight million, seven hundred sixty-five thousand, four hundred and twenty-two.

Note.—The teacher should vary the numbers, as may be necessary for practice.

Section 2. READING DECIMAL PARTS OF WHOLE NUMBERS.

As all whole numbers, or units, may be divided into any number of equal parts, it is important that all pupils should learn to read fractions, as well as whole numbers, and understand their meaning.

If a unit be divided into ten equal parts, the parts are called decimals, which means tenths.

If a straight line be divided into ten equal parts, thus, $|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|\frac{1}{10}|$, one part is called one tenth ($\frac{1}{10}$), and the whole line contains ten tenths; but if each of these tenths be divided into ten equal parts, the whole line will be divided into one hundred equal parts, and one of these smaller parts will be called one hundredth ($\frac{1}{100}$), etc. The usual way of expressing decimals is as follows: one tenth $=\frac{1}{10}=.1$. The period, or decimal point, must always be put immediately before the place for tenths. Five tenths $=\frac{5}{10}=.5$; ten tenths $=\frac{10}{10}=1$, or one unit. One hundredth $=\frac{1}{100}=.01$. Hundredths occupy the second place to the right of the decimal point; thousandths the third place, thus, five thousandths $=\frac{5}{1000}=.005$; ten-thousandths the fourth place, etc., thus, five ten-thousandths $=\frac{5}{10000}=.0005$; five hundred-thousandths $=\frac{5}{100000}=.00005$; and five millionths $=\frac{5}{1000000}=.000005$.

Test Examples for Reading.

.5	.55	.555	.5555	.55555	.555555
.9	.49	.625	.7845	.45625	.845675
.0	.09	.305	.7450	.06381	.706803
.05	2.025	.805	.00005	.8002	50.0050

Note.—The teacher must add and vary examples until the pupils can read readily and understandingly any decimal expression, no matter how long it may take.

Also, require the pupils to write the following literal numbers in Arabic characters or figures: Five tenths; sixty-five hundredths; one hundred and twenty-five thousandths; three thousand two hundred and forty-six ten-thousandths; eighty-three thousand six hundred fifty-seven hundred-thousandths; seven hundred and twenty-five thousand, three hundred and forty-five millionths; also, one hundred and five hundredths; ten thousand five hundred thousandths; fourteen hundred-thousandths and two millionths.

The test examples for reading, given above, should also be written in literal characters.

Exercises similar to the above should be kept up until the pupils can read or write any numbers, whole or decimal; but great care will be necessary to write all numbers legibly, and to write them so that those of the same name shall stand directly under one another.

These principles ought to be *taught orally*, before attempting to use this book. They are to be *taught* by the teacher and not memorized by pupils.

Section 3. READING COMMON, OR VULGAR, FRACTIONS.

A unit, as we have seen, can be divided into any other number of equal parts than ten, or decimals. When so divided the parts are called Common Fractions. These fractions are introduced here because they are wanted early, and because the *same principles* control all methods of operating with them which control operations in whole numbers and decimals.

Principles to be Kept in Mind.

1. All figures, as well as words, have a specific meaning, which must be recognized and kept in mind.

2. If different numbers are to be increased or diminished by other numbers, they must all be reduced to the same form and name, if necessary.

3. Decimal fractions are essentially the same as whole numbers, because ten parts of one denomination, or order of units, are required to make a unit of the next higher order.

But the parts of a *Common Fraction* are not always of the same value, nor of the same name, though they may be changed to the same name without changing their value. When fractions are thus changed, they can be used as whole numbers, by keeping in mind their name.

A failure to understand and recognize this property of fractions is the cause of most of the failures in learning to use fractional numbers.

We have seen that one unit is equal to ten tenths, so also one unit is equal to two halves ($\frac{2}{2}$); also to three thirds ($\frac{3}{3}$); also to four fourths ($\frac{4}{4}$); also to five fifths ($\frac{5}{5}$); and whenever the number of equal parts taken of anything is equal to the number of parts into which the thing is divided, the parts taken are equal to unity. (Illustrate by cutting an apple, or dividing a line on the blackboard.)

Reading Exercises.

(To be written on the blackboard.)

Two halves$=\frac{2}{2}=1$. Three thirds$=\frac{3}{3}=1$. Four fourths $=\frac{4}{4}=1$. Five fifths$=\frac{5}{5}=1$. Six sixths$=\frac{6}{6}=1$. Seven sevenths$=\frac{7}{7}=1$. Eight eighths$=\frac{8}{8}=1$. Nine ninths$=\frac{9}{9}$ $=1$. Ten tenths$=\frac{10}{10}=1$.

But two thirds ($\frac{2}{3}$) are less than three thirds ($\frac{3}{3}$), and are therefore less than unity; and as four thirds ($\frac{4}{3}$) are more than three thirds ($\frac{3}{3}$), they are more than unity. Again, four halves ($\frac{4}{2}$) are twice as much as two halves ($\frac{2}{2}$), and are therefore equal to two units (2).

These are essential preliminary steps in gaining a practical knowledge of fractions, but they should be repeated and varied by the teacher until the pupils can read understandingly any form of simple fractions.

Section 4. READING OF DENOMINATE NUMBERS.

Denominate numbers are such as require the name of the parts taken to be expressed. The common, or leading, unit is variously divided, and the parts used are expressed in whole numbers. If we had a decimal system of weights and measures, most of the operations in denominate numbers would be like those of whole and decimal numbers, and would thereby be much simplified. (See the article on the "Metric System," page 40.)

United States Money.

The measure of the different units in United States money is decimal, as follows: ten mills make one cent, ten cents make one dime, ten dimes make one dollar, and ten dollars make one eagle.

But in England and some of its provinces, money is reckoned in pounds, shillings, pence, and farthings; thus, four farthings make one penny, twelve pence make one shilling, and twenty shillings make one pound. Each country has its peculiar denominations of money. All the Tables of Denominate Numbers, in common use, will be given in the following pages, and they must be carefully memorized before the numbers can be used in arithmetical operations. As it is important that these tables should be familiar to every pupil, they are given here to be read and learned. The Tables of Denominations must all be memorized.

Note. — **See the Multiplication and Division Tables, pp. 1, 2.**

1. TABLES OF MONEY.

1. Federal Money.

10 Mills	make	1 Cent.
10 Cents	,,	1 Dime.
10 Dimes	,,	1 Dollar.
10 Dollars	,,	1 Eagle.

2. English Money.

4 Farthings (f.)	make	1 Penny.	— Sign,	d.
12 Pence	,,	1 Shilling.	,,	s.
20 Shillings	,,	1 Pound.	,,	£.
21 Shillings	,,	1 Guinea.	,,	g.

2. TABLES OF WEIGHT.

1. Avoirdupois Weight.

16 Drams (dr.)	make	1 Ounce.	— Sign,	oz.
16 Ounces	,,	1 Pound.	,,	lb.
25 Pounds	,,	1 Quarter.	,,	qr.
4 Quarters	,,	1 Hundredweight.		cwt.
20 Hundredweight	,,	1 Ton.		t.

2. Troy Wegiht.

24 Grains (gr.)	make	1 Pennyweight.	— Sign,	pwt.
20 Pennyweights	,,	1 Ounce.	,,	oz.
12 Ounces	,,	1 Pound.	,,	lb.

The Troy pound contains 5,760 grains; and the Avoirdupois pound contains 7,000 grains Troy.

3. Apothecaries' Weight.

20 Grains (gr.)	make	1 Scruple.	— Sign,	sc. or ℈.
3 Scruples	,,	1 Dram.	,,	dr. or ʒ.
8 Drams	,,	1 Ounce.	,,	oz. or ℥.
12 Ounces	,,	1 Pound.	,,	lb. or ℔.

3. TABLES OF EXTENSION.

1. Long Measure.

12 Inches (in.)	make	1 Foot.	— Sign,	ft.
3 Feet	,,	1 Yard.	,,	yd.
5½ Yards or 16½ Feet	,,	1 Rod.	,,	rd.
40 Rods	,,	1 Furlong.	,,	fur.
8 Furlongs	,,	1 Mile.	,,	mi.
3 Miles	,,	1 League.	,,	lea.

2. Surveyors' Measure.

7.92 Inches	make	1 Link.	— Sign,	li.
100 Links	,,	1 Chain.	,,	ch.
80 Chains	,,	1 Mile.	,,	mi.

3. Square Measure.

144 Square Inches	make	1 Square Foot.	— Sign,	sq. ft.
9 Square Feet	,,	1 Square Yard.	,,	sq. yd.
30¼ Square Yards	,,	1 Square Rod.	,,	sq. rd.
40 Square Rods	,,	1 Rood.	,,	R.
4 Roods	,,	1 Acre.	,,	A.
640 Acres	,,	1 Square Mile.	,,	sq. mi.

4. Land Measure.

10,000 Square Links (sq. li.)	make	1 Sq. Chain.	— Sign,	sq. ch.
10 Square Chains	,,	1 Acre.		A.
160 Square Rods	,,	1 Acre.		A.

5. Solid, or Cubic, Measure.

1,728 Cubic Inches	make	1 Cubic Foot.	— Sign,	cu. ft.
27 Cubic Feet	,,	1 Cubic Yard.	,,	cu. yd.
		ALSO,		
16 Cubic Feet	,,	1 Cord Foot.	,,	cd. ft.
8 Cord Feet	,,	1 Cord.	,,	cd.
128 Cubic Feet	,,	1 Cord.	,,	cd.

4. TABLES OF CAPACITY.

1. Dry Measure.

4 Gills	make	1 Pint.	— Sign,	pt.
2 Pints	,,	1 Quart.	,,	qt.
8 Quarts	,,	1 Peck.	,,	pk.
4 Pecks	,,	1 Bushel.	,,	bush.

The Winchester bushel contains 2,150.4 cubic inches, nearly, and is cylindrical, 18½ inches across and 8 inches deep.

2. Liquid Measure.

4 Gills (gi.)	make	1 Pint.	— Sign,	pt.
2 Pints	,,	1 Quart.	,,	qt.
4 Quarts	,,	1 Gallon.	,,	gal.
31½ Gallons	,,	1 Barrel.	,,	bbl.
63 Gallons	,,	1 Hogshead.	,,	hhd.
2 Hogsheads	,,	1 Pipe.	,,	pi.
2 Pipes	,,	1 Tun.	,,	tun.

The United States standard gallon contains 231 cubic inches, or 8.33888 pounds Avoirdupois, of distilled water.

5. TABLES FOR TIME.

60 Seconds (sec.)	make	1 Minute.	— Sign,	min.
60 Minutes	,,	1 Hour.	,,	hr.
24 Hours	,,	1 Day.	,,	da.
7 Days	,,	1 Week.	,,	wk.
365 Days	,,	1 Common Year.		yr.
366 Days	,,	1 Leap Year.		
100 Years	,,	1 Century.	,,	c.

The true length of a solar year is 365 days, 5 hours, 48 minutes, 47½ seconds.

Days in Each Month.

January	31	Days.	July	31	Days.
February	28 or 29	,,	August	31	,,
March	31	,,	September	30	,,
April	30	,,	October	31	,,
May	31	,,	November	30	,,
June	30	,,	December	31	,,

Once in four years February has 29 days.

6. CIRCULAR AND TIME MEASURE.

Longitudinal circles on the earth are so used as to measure distance around the earth and the difference in time between any two meridians. A new nomenclature is here given.

Table.

60 *Motes* (or seconds) (″)	make	1 *Mite* (or minute).	(′)
60 *Mites* (or minutes)	,,	1 Degree.	(°)
30 Degrees	,,	1 Sign.	(s.)
12 Signs or 360 Degrees	,,	1 Circumference.	(cir.)

Applied to Time.

15 Degrees in Longitude make		1 Hour in Time.
15 Mites (or minutes) in Longitude make		1 Minute in Time.
15 Motes (or seconds) in Longitude	,,	1 Second ,,
1 Degree in Longitude	,,	4 Minutes ,,
1 Mite (or minute) in Longitude	,,	4 Seconds ,,
1 Mote (or second) in Longitude	,,	$\frac{1}{15}$ Second ,,

NOTE 1. A circle is a plane figure bounded by a curved line, every point of which is equally distant from the centre of the circle.

2. A circumference of a circle is the line bounding the circle.

3. An arc is any part of the circumference.

4. A degree is $\frac{1}{360}$ part of the circumference.

5. A quadrant is one fourth of the circumference.

6. A sextant is one sixth of the circumference.

7. TABLES OF PARTICULARS.

12 Units	make	1 Dozen.		24 Sheets	make	1 Quire.
12 Dozen	,,	1 Gross.		20 Quires	,,	1 Ream.
12 Gross	,,	1 Great Gross.		2 Reams	,,	1 Bundle.
20 Units	,,	1 Score.		5 Bundles	,,	1 Bale.

MODES OF OPERATING IN NUMBERS.

Having now learned to read and understand the language of all kinds of numbers in common use, we are prepared to consider and learn the *four* different methods of operating in the four kinds of numbers.

1. All the different kinds of numbers can be added, and the process is called addition.

2. All kinds of numbers can be subtracted, or the difference between two may be found, and the process is called subtraction.

3. All kinds of numbers can be multiplied, or increased by repetition, which is called multiplication.

4. All kinds of numbers can be divided. The process is called division, or finding how many times one number is contained in another, or how many times one number can be subtracted from another.

CHAPTER II.

ADDITION OF ALL KINDS OF NUMBERS.

Section 1. WHOLE NUMBERS.

There are only *nine* significant characters, or figures, to be used in all purely arithmetical operations. They are 1, 2, 3, 4, 5, 6, 7, 8, 9. These characters make the alphabet of arithmetic. When the cipher (0) is used at the right of any digit, or of any number of digits, the value of the digit, or digits, if not a decimal, is increased ten times.

In adding numbers, one of these digits is always added to one or more digits of the same name, making the sum of the digits equal to the sum of the units represented by the digits.

The following exercises include all the possible additions of the digits, and should be memorized : —

Exercise 1.	*Exercise 2.*	*Exercise 3.*
$1+1=2$	$2+2=4$	$3+3=6$
$1+2=2+1=3$	$2+3=3+2=5$	$3+4=4+3=7$
$1+3=3+1=4$	$2+4=4+2=6$	$3+5=5+3=8$
$1+4=4+1=5$	$2+5=5+2=7$	$3+6=6+3=9$
$1+5=5+1=6$	$2+6=6+2=8$	$3+7=7+3=10$
$1+6=6+1=7$	$2+7=7+2=9$	$3+8=8+3=11$
$1+7=7+1=8$	$2+8=8+2=10$	$3+9=9+3=12$
$1+8=8+1=9$	$2+9=9+2=11$	
$1+9=9+1=10$		

Exercise 4.	*Exercise 5.*	*Exercise 6.*
4+4= 8	5+5=10	6+6=12
4+5=5+4= 9	5+6=6+5=11	6+7=7+6=13
4+6=6+4=10	5+7=7+5=12	6+8=8+6=14
4+7=7+4=11	5+8=8+5=13	6+9=9+6=15
4+8=8+4=12	5+9=9+5=14	
4+9=9+4=13		

Exercise 7.	*Exercise 8.*	*Exercise 9.*
7+7=14	8+8=16	9+9=18
7+8=8+7=15	8+9=9+8=17	
7+9=9+7=16		

These exercises should be repeated and varied until each of these combinations can be made as quickly as seen or heard. Every teacher should make a careful study of Grube's methods.*

Methods of Securing Accuracy and Rapidity.

1. ORAL AND MENTAL EXERCISES.

1. The teacher may name any two numbers, and teach the pupils to give the sum, and in the same way teach them to combine each of the numbers taken with all the other digits.

3+4=7. Then 3+2, 3+3, 3+4, 3+5, 3+6, 3+7, 3+8, 3+9. Then 4+1, 4+2, 4+3, 4+4, 4+5, 4+6, etc.

2. The teacher will next take any three digits, and require accurate and quick answers; and then combine any of the three digits thus taken with any two of the remaining digits.

3+4+5=12. Then, 3+1+2, 3+6+7, 3+8+9. Then, 4+1+2, 4+5+6, 4+3+7, 4+8+9. Then, 5+1+2=8. 5+3+4, 5+5+6, 5+7+8, 5+8+9.

* "Grube's Method of Teaching Primary Arithmetic." By F. Louis Soldan. Published by S. R. Winchell & Co., Chicago. Price, 30 cents.

3. Next take four numbers or digits and combine each with any three others.

3+2+1+4=10; 3+5+6+7=21; and so on.

Note. — In all cases, results should be given as soon as possible, without counting.

4. Next take five numbers and use them in the same manner.

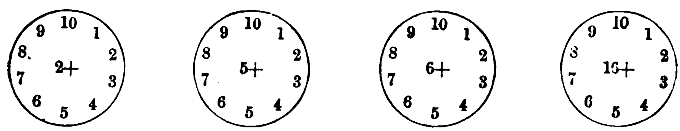
2+1+3+4+5=15, etc.

5. Next take six digits and find their sum as above.

1+2+3+4+5+6=21; and so on.

These exercises can be varied, until the pupils are able to give *accurate* and *quick* results, by using any number of digits repeated. They should be kept up daily and regularly, not only to teach pupils to *add* accurately and rapidly, but as a help to receive ideas accurately and rapidly.

2. SIGHT AND MENTAL EXERCISES.

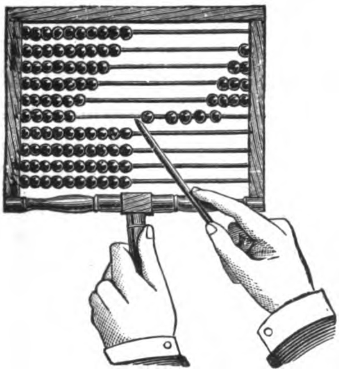


The teacher will stand before one of these diagrams neatly drawn upon the blackboard, and explain the work to be done by telling each member of the class to keep in mind the number in the centre, and add it, as the sign requires, to any number to which the teacher may point, going as rapidly from one number to another as possible, and securing accurate answers. If different answers are given, the teacher must be sure that all get the right answer before leaving the number. *Secure accuracy first.*

The number in the centre, as well as those in the circumference, of the circle may be exchanged for any other numbers.

Any amount of *sight*-adding may be secured by the use of this diagram, which serves to give just the training needed for accurate and rapid additions.

3. THE NUMERAL FRAME.



The value of this simple piece of apparatus is not generally appreciated or properly understood by elementary teachers as a means of imparting a correct idea of numbers and of teaching the first principles of the four arithmetical operations: addition, subtraction, multiplication, and division.

By the proper use of this frame, children can be taught what *counting* actually is; that it is not simply repeating the names of the common numbers in their order — a common, but erroneous, practice. It may be well for children to learn to count by *twos*, *threes*, *fours*, and so on, forward and backward; but not for the sake of learning to add and subtract.

This frame is not to be used as a simple *plaything;* but every movement of balls should be for a specific purpose, and should help the pupils to learn that numbers represent things just as words represent ideas.

Every elementary teacher should study this frame, and learn all its valuable uses.

4. USE OF BEANS, BLOCKS, ETC.

As a supplementary means of giving correct instruction in the first principles of numbers, beans, blocks, etc., may be used successfully by way of variety. Each pupil may be supplied with fifty or one hundred of these, or with kindergarten splints; but they must never be used aimlessly, but always for the purpose of impressing some idea, for which every teacher should be specially trained.

Section 2. ADDITION OF COMBINED DIGITS.

You have already learned that when whole numbers or digits are combined in proper order, the place for the first order is called the *unit's place*, and the next place to the left is called the *ten's place*, because any figure in the ten's place is ten times as great in value as when it is in the unit's place; and the same figure removed one place further to the left, or to hundred's place, is ten times as great in value as it is in the ten's place, and one hundred times greater than when it is in the unit's place; and so on, increasing ten times for each removal.

The sum or amount of several numbers is the result of adding them together. When they are to be added the numbers must be so written that those of the same name will stand directly under each other, namely: units under units, tens under tens, hundreds under hundreds, pounds under pounds, shillings under shillings, etc. etc. Thus, if 35 is to be added to 53, the numbers will be prepared for addition by writing them thus:* $\begin{array}{r} 35+ \\ 53 \\ \hline \end{array}$;

and 357 added to 464, will be placed thus: $\begin{array}{r} 357+ \\ 464 \\ \hline \end{array}$

For convenience' sake we begin to add the figures of the lowest value first, and proceed in order to figures of the next higher value.

* The signs are expressed here, first, to show what is to be done; second, to make pupils familiar with them.

Take, for illustration, the first example given above: three units (3) added to five units (5) make 8 units, which is to be written under units; and five tens (5) added to three tens (3) make 8 tens, which is to be written under tens.

t.	u
3	5+
5	3
8	8

357+
464
821

In the second example given above, four units (4) added to seven units (7) make eleven units (11), which is one ten and one unit. Put the one unit (1) in the place of units, and the one ten (1) added to the six tens (6) and the five tens (5), thus, 1+6+5, equal twelve tens (12), which is one hundred (1) and two tens (2). The 2 must be placed, of course, in the ten's place, and the one hundred added to the four hundreds and three hundreds, thus, 1+4+3=eight hundreds, in the hundred's place.

Table of Integral Numbers.

10 Units	make	1 Ten.
10 Tens	,,	1 Hundred.
10 Hundreds	,,	1 Thousand.
10 Thousands	,,	1 Ten-thousand.
10 Ten-thousands	,,	1 Hundred-thousand.
10 Hundred-thousands	,,	1 Million.

So always, ten of any denomination, or order of units, make one of the next higher order of units.

Examples for Writing and Adding.

1.	2.	3.	4.	5.
				45+
45+	76+	452+	768+	53
53	57	534	574	61
98	133	986	1342	159

6.	7.	8.	9.
		765+	768+
765+	452+	576	574
576	534	753	625
753	613	646	876
2094	1599	2740	

10.	11.	12.
1234+	12345+	1234567+
2345	23456	2345678
3456	34567	3456789
4567	45678	4567891
5678	56789	5678912

The above examples are sufficient to illustrate all the principles which control the operations in addition of whole numbers. But to secure accuracy and rapidity in practice, the teacher can easily supply any number of similar examples. The number of figures in each column may be increased to any desirable extent, and should be, of course.

A model test example. Add from bottom to top, and from right to left.

No. 1.

123456789	45
987654321	45
234567891	45
198765432	45
345678912	45
219876543	45
456789123	45
321987654	45
567891234	45
3456667899	

No. 2.

						Amounts.
	25	125	55	321	1322	1848
	36	432	68	465	8467	9468
	42	265	75	943	5632	6957
	35	734	92	728	7564	9153
	28	185	45	564	3482	4304
	81	891	99	222	2677	3970
Amts.	247	2632	434	3243	29144	35700

Similar examples to these may be given for practice. The last addition proves the accuracy of the work.

Section 3. ADDITION OF DECIMAL NUMBERS.

Decimal numbers are really fractions, or the equal parts of whole numbers which have been divided into tenths. One of these tenths may be divided, or supposed to be divided, into ten other equal parts, which take the name of hundredths. (See Sec. 2, Chap. I.) Decimal places decrease in value tenfold for every removal to the right.

When decimals are used, the first figure, or place of tenths, is the next figure at the right of units. Immediately before this first decimal figure should always be placed a decimal point, called *separator*, like the common period; thus 5.5 is the same as five whole numbers and five tenths; or 45.7 is the same as forty-five and seven tenths.

The second figure to the right of the separator is called hundredths; thus 5.55 is the same as five whole numbers and fifty-five hundredths; and the third figure is thousandths; thus 5.625 is the same as five whole numbers and six hundred and twenty-five thousandths.

The same digits are used as signs of the decimal parts which are used to express whole numbers; so that the work of adding them is essentially the same as that of adding whole numbers.

In writing decimal numbers to be added, figures of the same name must be placed directly under each other, or in the place indicated by their name. These operations must be thoroughly understood.

Examples for Adding Decimals.

Exercise 1.	*Exercise 2.*	*Exercise 3.*
.3	.75	.0025
3.33	25.25	1.0001
.025	75.075	10.00000
.625	.625	101.100001
5.825	5.875	7.360004
10.105		

The teacher must supply a sufficient number of similar examples to make all the pupils understand how to write and add decimals.

Exercise 4.

Add five tenths, fifty-six hundredths, six hundred and twenty-five thousandths, sixteen thousandths, eight hundred-thousandths, twenty-five millionths, to one millionth.

Exercise 5.

Add fifteen, and fifteen thousandths; one hundred and five, and thirty-seven ten-thousandths; six hundred and twenty-five thousand, three hundred and twenty-one millionths; to one hundred and one hundred-thousandths.

Exercise 6.

Add three hundred and twenty-five thousandths; sixteen, and sixteen thousandths; eight hundred and seventy-five thousandths; twenty-eight hundredths; one hundred and twenty-five hundredths; to one thousand two hundred and eighty-five millionths.

The teacher should add other similar examples for practice.

Section 4. ADDITION OF COMMON FRACTIONS.

We have found that fractions in a decimal form can be added; we must also learn how to add common fractions in different forms, such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, etc. The number of equal parts into which any unit is divided always gives the name to the fraction, and it is called the denominator or *namer*. In writing the fraction, the denominator is usually below a short line, and is to be used as a divisor.

The number showing how many parts of the fraction are used is called the numerator, and is placed above the line, over the denominator. (See Sec. 3, Chap. I.)

We have learned from the *second fundamental principle* (page 12, Chap. I) that if numbers to be compared, or added and subtracted, are not of the same name and form, they must be made so before they can be added or subtracted.

ILLUSTRATION.—If one unit is divided into *two* equal parts, and another unit of the same kind is divided into *three* equal parts, the parts can be reduced to the same name or value by finding the smallest number which can be equally divided into *halves* or *thirds*.

This can be done either by inspection or by multiplying the denominators together for a new denominator, and then by taking such a part of this new denominator, for a new numerator, as the value of the given fraction requires.

Thus, to add $\frac{1}{2}$ and $\frac{1}{3}$, we can get these fractions to the same name by taking 6 for the denominator of each; and *one half* of $6=3$, for the first numerator; and *one third* of $6=2$, for the second numerator, which will make the fractions $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$, the sum of $\frac{1}{2}+\frac{1}{3}$.

Again, to add $\frac{1}{2}$ and $\frac{2}{3}$ and $\frac{3}{4}$, we can find easily, by inspection, the smallest number to be twelve of which we can find a half, a third, and a fourth. Twelve, then, is the common denominator; and $\frac{1}{2}$ of $12=6$ is the first new numerator; $\frac{2}{3}$ of $12=8$ is the second new numerator; and $\frac{3}{4}$ of $12=9$ is the third new numerator$=\frac{6}{12}+\frac{8}{12}+\frac{9}{12}=\frac{23}{12}=1\frac{11}{12}$.

Note. — The multiplication and division tables must be thoroughly memorized, so that every pupil may be able to think at once how many times any one of the nine digits is contained in the product of itself by any other digit. Thus, how many times is 4 contained in 8, 12, 16, 20, etc.?

Rule for Changing Fractions.

To reduce fractions of different names or denominations to equivalent fractions of the same name.

RULE. — *Find, by inspection or trial, the smallest number that will contain each of the denominators of the given fractions an exact number of times, and that number will be the least common multiple or denominator of the required value. Take such a part of this multiple as is indicated by each of the fractions given, for its new numerator.*

Or, if it is necessary to use the common rule for finding the Least Common Multiple, write the denominators in a line and divide them by the largest *prime number* that will exactly divide *two* or more of them, and write the quotients and undivided numbers in a line again and divide as before, and so continue until no prime number will exactly divide two of them; then multiply the divisors, quotients, and undivided numbers together for the smallest common denominator. Then, as by the rule of inspection, find the numerators by taking such a part of this denominator as is indicated by each given fraction.

A *prime* number is one which cannot be resolved into even factors.

Note.—The Common Multiple of two or more fractions may be found more to gratify curiosity than to serve any practical use. RULE.—*Reduce all the fractions to equivalent fractions having a common denominator, and then find the Common Multiple of the numerators and write the result over the Common Denominator.*

Before attempting any operations under these rules, if all the fractions to be added are not of the same denomination, or if there are any compound or complex fractions, they must first be reduced to the same denomination, or parts of the same kind of unit; and second, all compound fractions and all complex fractions must be reduced to *simple* fractions. For example, $\frac{1}{2}$ of a pound and $\frac{2}{3}$ of a pound are fractions of the same denomination, namely, pounds; but $\frac{1}{2}$ of a pound (Sterling money) and $\frac{2}{3}$ of a shilling are of different denominations, and before they can be added they must be changed to the same denomination, thus: as a pound is 20 shillings, half of a pound is 10 shillings. We can now add 10s. and $\frac{2}{3}$ of a shilling, making $10\frac{2}{3}$ shillings.

Compound Fractions.

But one half of two thirds of a pound, usually expressed $\frac{1}{2}\times\frac{2}{3}$ of a pound, is a compound fraction. We must find one half of two thirds, which is one third, or $\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$, and $\frac{1}{3}$ is a simple fraction.

2. Again, $\frac{2}{3}$ of $\frac{4}{5}$ is a compound fraction, and there are two ways of making it a simple fraction.

First, we will find the common denominator by the rule of inspection (page 30), which is 15; and $\frac{4}{5}=\frac{12}{15}$, and now $\frac{2}{3}\times\frac{12}{15}=\frac{8}{15}$, the simple fraction.

Second, all compound fractions can be made simple by multiplying all the numerators together for a new numerator and all the denominators together for a new denominator thus, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

Complex Fractions.

But *four* divided by *one half* (or $\frac{4}{\frac{1}{2}}$) is a *complex fraction*. To make this fraction simple we must first reduce 4 to halves. This gives us eight halves ($\frac{8}{2}$), and one half is contained eight times in eight halves.

Again $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ is a complex fraction. To simplify this fraction and all similar fractions, change the numerator ($2\frac{1}{2}$) to an improper fraction, thus, $2\frac{1}{2} = \frac{5}{2}$, and then change the denominator ($3\frac{1}{3}$) also to an improper fraction, thus, $3\frac{1}{3} = \frac{10}{3}$. Now see how many times $\frac{10}{3}$ is contained in $\frac{5}{2}$.

First reduce the fractions to the same name, which will be sixths (6) ; thus, $\frac{10}{3} = \frac{20}{6}$, and $\frac{5}{2} = \frac{15}{6}$. The question requires $\frac{15}{6}$ to be divided by $\frac{20}{6}$. The denominators being the same, they may be disregarded, and the numerator 15 must be divided by the numerator 20, making the simple fraction $\frac{15}{20} = \frac{3}{4}$.

Note.—These methods of simplifying fractions must be thoroughly learned.

Examples of Fractions to be Added.

1. Add $\frac{2}{3}$ and $\frac{3}{4}$.

ILLUSTRATION. — (1) Reduce to the same name, or to a common denominator, which is 12. (2) Take $\frac{2}{3} \times 12 = 8$, for the first numerator; and take $\frac{3}{4} \times 12 = 9$, for the second numerator. (3) Add the numerators $\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12} =$ ANS.

2. Add $\frac{3}{4}$ and $\frac{4}{5}$.

Proceed as with the last example. The least common denominator is 20, and $\frac{3}{4}$ of $20 = 15$ is the first numerator, and $\frac{4}{5}\times 20=16$ is the second numerator, thus: $\frac{15}{20}+\frac{16}{20}=\frac{31}{20}=1\frac{11}{20}$ = Ans.

3. Add $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{4}$.

By trial we find the least common denominator is 12. The numerators are $\frac{1}{2}\times 12=6$; $\frac{1}{3}\times 12=4$, and $\frac{1}{4}\times 12=3$. The fractions then added become $\frac{6}{12}+\frac{4}{12}+\frac{3}{12}=\frac{13}{12}=1\frac{1}{12}$ = Ans.

4. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

We find the least common denominator to be 60; then $\frac{1}{2}\times 60=30$, $\frac{2}{3}\times 60=40$, $\frac{3}{4}\times 60=45$, and $\frac{4}{5}\times 60=48$. $\frac{30}{60}+\frac{40}{60}+\frac{45}{60}+\frac{48}{60}=\frac{163}{60}=2\frac{43}{60}$ = Ans.

5. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.

The least common denominator is 60. The fractions are then $\frac{30}{60}+\frac{40}{60}+\frac{45}{60}+\frac{48}{60}+\frac{50}{60}=\frac{213}{60}=3\frac{33}{60}=3\frac{11}{20}$ = Ans.

6. Add $\frac{2}{3}$ and $\frac{3}{7}$.

The least common denominator is 21, and the fractions then are $\frac{2}{3}\times 21=\frac{14}{21}$, and $\frac{3}{7}\times 21=\frac{9}{21}=\frac{14}{21}+\frac{9}{21}=\frac{23}{21}=1\frac{2}{21}$ = Ans.

7. Add $\frac{5}{6}$ and $\frac{5}{7}$.

The least common denominator is 42, and the fractions are now $\frac{35}{42}+\frac{30}{42}=\frac{65}{42}=1\frac{23}{42}$ = Ans.

8. Add $\frac{6}{7}$, $\frac{3}{8}$, and $\frac{5}{6}$.

To find the least common denominator, take out the common factor 2, in the denominators 6 and 8, and the product of the remaining factors $7\times 4\times 3\times 2=168$, the least common multiple. The new numerators are $\frac{6}{7}\times 168$, $\frac{3}{8}\times 168$, and $\frac{5}{6}\times 168$, making the fractions $\frac{144}{168}+\frac{63}{168}+\frac{140}{168}=\frac{347}{168}=2\frac{11}{168}$ = Ans.

9. Add $\frac{3}{8}$, $\frac{2}{9}$, and $\frac{5}{11}$.

Here, there is no common factor of the denominators but 1; so the common denominator is the product of all the denominators, which is 792. The numerators are found as above.

10. Add $4\frac{1}{2}+\frac{2}{3}+\frac{5}{6}$.

From the previous illustrations it must be evident that 6 is the common denominator; and the terms of the example must be $4\frac{3}{6}+\frac{4}{6}+\frac{5}{6}=4+\frac{12}{6}=4+2=6=$ ANS.

11. Add $5\frac{1}{2}$, $8\frac{1}{3}$, and $7\frac{1}{5}$.

Here the least common denominator is 30; and the terms will be $5\frac{15}{30}+8\frac{10}{30}+7\frac{6}{30}=20+\frac{31}{30}=21\frac{1}{30}=$ ANS.

Mixed numbers are whole numbers united with fractions, and can be reduced to improper fractions by multiplying the whole number by the denominator, adding the numerator to the product, and writing the result over the denominator; thus, $6\frac{2}{3}=\frac{3\times 6+2}{3}=\frac{20}{3}$.

Note. — Generally, when mixed numbers are to be added, the whole numbers should first be added, and then the fractions added separately, as in previous cases, and then their value added to the sum of the whole numbers.

Again, *a Proper fraction* has the numerator *less* than the denominator, and is therefore less than unity.

An Improper fraction has a numerator equal to, or greater than, the denominator, and is equal to, or more than, unity.

Again, if the numerator of an improper fraction be divided by the denominator, the quotient will be a whole or a mixed number; thus, $\frac{17}{3}=5\frac{2}{3}$.

Although the previous examples and some of the principles seem to anticipate the rules of multiplication and division, they do not require any operation which is not taught by the Tables of Multiplication and Division, given at the beginning, which must be thoroughly mastered at first.

12. Find the sum of $4\frac{1}{2}+\frac{2}{3}+\frac{5}{6}+2\frac{1}{5}$.

13. Find the sum of $\frac{3}{7}+\frac{5}{9}+\frac{4}{11}$.

14. Find the sum of $\frac{3}{7}+\frac{5}{9}+\frac{4}{14}$.

15. Find the sum of $\frac{2}{7}+\frac{3}{14}+\frac{9}{28}$.

16. Find the sum of $\frac{1}{3}+\frac{1}{2}\times\frac{3}{4}+\frac{\frac{1}{2}}{\frac{2}{3}}$.

17. Find the sum of $\frac{3}{4}+\frac{2}{3}+\frac{4}{5}+\frac{2\frac{1}{2}}{3}$.

18. Find the sum of $6\frac{1}{4}+\frac{1}{6}\times\frac{5}{6}+\frac{11}{5\frac{1}{2}}$.

19. What part of a dollar is the sum of one half, one third, one fourth, one sixth, one eighth, and one twelfth, of a dollar?

Section 5. ADDITION OF DENOMINATE NUMBERS.

Denominate numbers may be whole numbers or fractions; but they are always used in connection with the name of the whole thing, or of its parts.

The numbers are variously, but arbitrarily, divided into subordinate equal parts.

Before attempting to operate in denominate numbers, the pupils must review the tables, given in Chapter I, and have them thoroughly memorized.

As in writing whole numbers and decimals for addition, special care must be taken to place denominations of the same name directly under each other.

RULE. — *When all the units of any denomination are added, as in whole or decimal numbers, divide the sum by the number given in the tables which is required of this denomination to make a unit in the next higher denomination; add the quotient to the sum of the numbers of the next higher denomination, and divide by the* table number *of this denomination as before, and place the remainder, if any, under the denomination* *added, and thus proceed until the highest denomination is added.*

This rule must be thoroughly understood.

Examples in Denominate Numbers.

1. FEDERAL, OR UNITED STATES, MONEY.

1.

	$	d.	cts.	m.
Add,	5	6	5	8
	12	5	8	5
	25	6	2	5
	43	8	6	8

2.

	$	d.	cts.	m.
Add,	25	5	6	3
	7	3	5	8
	15	7	4	7

3.

$525.275
130.87
461.875
5.29
.875

4.

$2002.083
1055.17
625.105
3581.125
15495.95

5. Write in figures and find the amount of five dollars and twenty-five cents; eighty-seven cents and five mills; forty-six dollars, sixty-two cents, and five mills; one hundred and fifteen dollars, eighty-seven and a half cents; and six dollars and six cents.

6. Find, also, the amount of six and a quarter cents; one dollar, thirty-seven and a half cents; four eagles, six dollars, seven dimes, and five mills; seventy dollars, five dimes, five cents, and five mills; six thousand, three hundred eighty-seven dollars, and ninety-seven cents; twenty-five eagles, fifteen dollars, nine dimes, eight cents, and five mills.

2. ENGLISH MONEY.

1.

	£	s.	d.	far.
Add,	5	6	8	2
	21	12	9	3
	15	15	11	2
	42	15	5	3

2.

	£	s.	d.	far.
Add,	21	8	7	2
	15	7	3	0
	21	12	9	3
	32	18	8	1

3. Add £2, 2s., 6d.; £115, 10s., 6d., 3far.; £125, 13s., 6d.; £28, 14s., 10d., 3far.; 15s., 8d., 2far.; and £19, 18s., 11d., 2far.

4. Add twenty pounds (£), eighteen shillings (s.), ten pence (d.), two farthings (far.); five pounds, twelve shillings, five pence, three farthings; and twenty pounds, nine shillings, three pence, two farthings.

3. AVOIRDUPOIS WEIGHT.

1.

	T.	cwt.	qr.	lb.	oz.	dr.
Add,	3	2	3	20	12	5
	1	19	2	15	8	13
			3	24	15	15
	5	3	2	11	5	1

2.

	T.	cwt.	qr.	lb.	oz.	dr.
Add,	4	15	3	16	8	0
	3	11	2	23	7	3
		17	1	17	15	15

3. Add, 6cwt., 2qr., 15lb.; 3qr., 18lb., 11oz., 9dr.; 2T., 3qr., 18lb., 13oz.; and 9cwt., 3qr., 18lb., 13oz.

4. TROY WEIGHT.

1.

	lb.	oz.	pwt.	gr.
Add,	15	10	18	21
	26	8	15	23
	7	11	10	20
	50	7	5	16

2.

	lb.	oz.	pwt.	gr.
Add,	5	11	15	9
	15	9	18	3
	1	3	11	21

3. Add, 9lb., 10oz., 15pwt., 5gr.; 15lb., 8oz., 9pwt.; 1lb., 2oz.; 18lb., 10oz., 19pwt., 21gr.; and 125lb., 5oz., 15pwt., 12gr.

5. APOTHECARIES' WEIGHT.

1.

	lb.	℥	ʒ	℈	gr.
Add,	6	9	5	2	15
	1	10	4	1	12
	15	11	7	2	5
	24	8	2	0	12

2.

	lb.	oz.	dr.	sc.	gr.
Add,	9	9	7	2	10
	15	10	5	1	15
		11	6	2	19

3. Add five pounds, three ounces, two drams, one scruple, ten grains; fourteen pounds, nine ounces, six drams; and twenty-one pounds, five ounces, six drams, two scruples.

Note. — The signs may be ℔. oz. dr. scr. gr., if preferred.

6. LONG MEASURE.

1.

	mi.	fur.	rd.	yd.	ft.	in.
Add,	2	5	21	3	2	8
	5	6	32	4	1	9
	15	7	36	5	2	11
	24	4	11	3	1	4

2.

	mi.	fur.	rd.	yd.	ft.
Add,	55	3	25	2	0
	146	2	36	0	2
	226	5	39	1	1

3. Add six miles, two furlongs, thirty rods, three yards, two feet, nine inches; four miles, five furlongs, thirty-six rods, four yards, one foot, eleven inches; twelve miles, five furlongs, twenty-six rods, four yards, two feet, ten inches; twenty miles, thirty-seven rods, five yards, two feet, nine inches; and fifty miles, three furlongs, twenty-eight rods, three yards, two feet, eleven inches.

7. SQUARE MEASURE.

1.

	A.	R.	sq. rd.	sq. yd.	sq. ft.
Add,	4	3	22	21	6
	2	3	36	18	7
	5	2	25	28	5

2.

	A.	R.	sq. rd.	sq. yd.
Add,	56	3	35	25
	144	2	30	18
	25	1	18	20

8. CUBIC MEASURE.

1.

	cu. yd.	cu. ft.	cu. in.
Add,	15	21	468
	25	25	1240
	6	18	562

2.

	cd.	cd. ft.	cu. ft.
Add,	5	3	12
	7	7	10
	3	5	15

9. DRY MEASURE.

1.

	bu.	pk.	qt.	pt.
Add,	5	3	6	1
	6	2	7	1
	7	3	5	0

2.

	bu.	pk.	qt.
Add,	320	2	6
	$5\frac{1}{2}$	0	0
	20	3	7

10. LIQUID MEASURE.

1.

	hhd.	gal.	qt.	pt.	gi.
Add,	3	41	3	1	3
	8	53	2	1	2
	15	55	1	1	3

2.

	bbl.	gal.	qt.	pt.	gi.
Add,	5	28	3	1	2
	7	30	2	1	3
	10	16	1	1	1

11. MEASURE OF TIME.

1.

	c.	yr.	da.	hr.	mi.	sec.
Add,	2	50	225	18	55	45
	3	75	125	7	40	50
	5	81	250	15	30	40

2.

	w.	d.	h.	mi.	sec.
Add,	6	6	18	45	55
	3	5	20	50	40
	7	6	15	42	50

3. Add two years, eight months, twenty days; eighteen years, seven months, fifteen days; and twenty-five years, eleven months, twelve days.

12. CIRCULAR AND TIME MEASURE.

1.

	s.			
Add,	5	22°	51′	52″
	6	24	46	55
	9	25	45	56

2.

	s.			
Add,		23°	21′	52″
	1	14	53	46
	3	28	40	50

THE METRIC SYSTEM.

The Metric System, based upon the decimal scale, is destined to be universally adopted.

The base of the system is the Meter, which is the *one ten-millionth* part of the meridian distance on the earth's surface from the equator to the pole; or 39.37079 inches. The other measures, such as the *Are* (air), the *Stere* (stair), the *Liter* (leeter), and the *Gram*, are made from the Meter. These are the primary units of the system, from which all the tables are derived.

The Congress of the United States legalized this system July 28, 1866, making the Meter equal to 39.37 inches, which is about 1.09361 yards. The Square Meter was declared to be equal to 1.196 square yards; the Cubic Meter equals 1.308 cubic yards; the Liter equals .908 of a quart, dry measure, or 1.0567 liquid quarts, and the Kilogram equals 2.2046 avoirdupois pounds.

These values are nearly exact, but all changes of weights and measures from the common to the metric system, and the reverse, must be made in accordance with these values, in order to be legal.

The multiple divisions of the metrical unit are named by prefixing to the names of the primary units the Greek numerals Deka (10), Hecto (100), Kilo (1,000), and Myra (10,000).

The sub-multiple units, or lower denominations, are named by prefixing to the names of the primary units the Latin numerals, Deci ($\frac{1}{10}$), Centi ($\frac{1}{100}$), Mille ($\frac{1}{1000}$). These names of metric subdivisions correspond to the common terms, known as *whole numbers* and *decimals*.

1. MEASURES OF EXTENSION.

The Meter is the unit of length, and is 39.37 inches, nearly.

		1 Millimeter=.03937079 in.
10 Millimeters (m. m.)	=	1 Centimeter=.3937079 ,,
10 Centimeters (c. m.)	=	1 Decimeter=3.937079 ,,
10 Decimeters (d. m.)	=	1 METER =39.37079 ,,
10 Meters (m.)	=	1 Dekameter=32.808992 ft.
10 Dekameters (d. m.)	=	1 Hectometer=19.927817 rd.
10 Hectometers (h. m.)	=	1 Kilometer=.6213824 mi.
10 Kilometers (k. m.)	=	1 Myrameter=6.213824 mi.

The *Meter*, like our yard, is used in measuring cloths and short distances. The *Kilometer* is commonly used for long distances, and is about $\frac{5}{8}$ of a mile.

2. MEASURE OF SURFACE.

The *Are* (ar.) is the unit of land measure, and is a square whose side is 10 meters, equal to a *square dekameter*, or 19.6 square yards.

1 Centare (c. a.)	=	1 Square Meter=1.196034 sq. yd.
100 Centares (c. a.)	=	1 ARE=119.6034 sq. yd.
100 Ares (a.)	=	1 Hectare (h. t.)=2.47114 acres.

The *Square Meter* is the unit measure for ordinary surfaces, as floorings, ceilings, etc.

100 Sq. Millimeters (sq. m. m.)	=1 Sq. Centimeter	= .155 sq. in.
100 Sq. Centimeters (sq. c. m.)	=1 Sq. Decimeter	= 15.5 sq. in.
100 Sq. Decimeter (sq. d. m.)	=1 Sq. METER	= 1.196+ sq. yd.

The *Stere* is the unit of wood or solid measure, and is equal to a Cubic Meter, or .2759 of a cord.

	1 Decistere	=3.531+ cu. ft.
10 Decisteres (dst.)	=1 STERE	=35.313+ cu. ft.
10 Steres (st.)	=1 Dekastere	=13.079+ cu. yd.

The *Cubic Meter* is the unit for measuring ordinary solids, excavations, embankments, etc.

1,000 Cu. Millimeters (cu. m.m.)	=1 Cu. Centimeter	= .061+ cu. in.
1,000 Cu. Centimeters (cu. c. m.)	=1 Cu. Decimeter	= 61.026+ ,,
1,000 Cu. Decimeters (cu. d.m.)	=1 Cu. METER	= 35.316+ cu. ft.

3. MEASURES OF CAPACITY.

The Liter is the unit of capacity, both of Liquid and of Dry Measure; and is a vessel whose volume is equal to a Cube whose edge is $\frac{1}{10}$ of a Meter; equal to 1.05673 qts. Liquid Measure, and .9081 qt. Dry Measure.

10 Milliliters (mil.)	=1 Centiliter.
10 Centiliters (c. l.)	=1 Deciliter.
10 Deciliters (d. l.)	=1 LITER.
10 Liters (L.)	=1 Dekaliter.
10 Dekaliters (D.l.)	=1 Hectoliter.
10 Hectoliters (H. l.)	=1 Kiloliter.
10 Kiloliters (K. l.)	=1 Myrialiter (M. l.).

The Hectoliter is the unit in measuring liquids, grain, fruit, and roots, in large quantities.

Equivalents in U. S. Measures.

Metric Denominations.	Cubic Measure.	Dry Measure.	Wine Measure.
1 Myrialiter	= 10 Cubic Meters	= 283.72 bu.	= 2641.4+ gal.
1 Kiloliter	= 1 Cubic Meter	= 28.372 bu.	= 264.17 gal.
1 Hectoliter	= $\frac{1}{10}$ Cubic Meter	= 2.8372 bu.	= 26.417 gal.
1 Dekaliter	= 10 Cu. Decimeters	= 9.08 qts.	= 2.6417 gal.
1 Liter	= 1 Cu. Decimeter	= .908 qts.	= 1.0567 qts.
1 Deciliter	= $\frac{1}{10}$ Cu. Decimeter	= 6.1022 cu. in.	= .845 gi.
1 Centiliter	= 10 Cu. Centimeters	= .6102 cu. in.	= .338 fl. oz.
1 Milliliter	= 1 Cu. Centimeter	= .061 cu. in.	= .27 fl. dr.

4. MEASURES OF WEIGHT.

The Gram is the unit of weight, and is equal to the weight of a cube of distilled water, the edge of which is one hundredth of a meter, equal to 15.432 Troy grains.

10 Milligrams (m. g.)	= 1 Centigram	= .15432+ gr.	Troy.
10 Centigrams (c. g.)	= 1 Decigram	= 1.54324+ gr.	,,
10 Decigrams (d. g.)	= 1 Gram	= 15.43248+ gr.	,,
10 Grams (G.)	= 1 Dekagram	= .35273+ oz.	Avoir.
10 Dekagrams (D. g.)	= 1 Hectogram	= 3.52739 ,,	,,
10 Hectograms (H. g.)	= 1 Kilogram	= 2.20462+ lb.	,,
10 Kilograms (K. g.)	= 1 Myriagram	= 22.04621+ lb.	,,
10 Myriagrams or 100 Kilograms (M.g.)	= 1 Quintal	= 220.46212+ lb.	,,
1,000 Kilos or 10 Quintals	= 1 Tonneau or Ton.	= 2204.62125 lb.	,,

The Kilogram, or Kilo, is the unit of common weight in trade, and is a trifle less than $2\frac{1}{5}$ lbs. Avoirdupois.

The tonneau is used for weighing very heavy articles, and is about 204 lbs. more than a common ton.

Units of the common system may be readily changed to units of the *metric* system by the aid of the following table: —

1 Inch	=2.54 Centimeters.
1 Foot	=30.48 Centimeters.
1 Yard	=.9144 Meter.
1 Rod	=5.029 Meters.
1 Mile	=1.6093 Kilometers.
1 Square Inch	=6.4528 Square Centimeters.
1 Square Foot	=929 Square Centimeters.
1 Square Yard	=.8361 Square Meters.
1 Square Rod	=25.29 Centiares.
1 Acre	=40.47 Ares.
1 Square Mile	=259 Hectares.
1 Cubic Inch	=16.39 Cubic Centimeters.
1 Cubic Foot	=28,320 Cubic Centimeters.
1 Cubic Yard	=.7646 Cubic Meter.
1 Cord	=3.625 Steres.
1 Fluid Ounce	=2.958 Centiliters.
1 Gallon	=3.786 Liters.
1 Bushel	=.3524 Hectoliter.
1 Troy Grain	=64.8 Milligrams.
1 Troy Pound	=.373 Kilo.
1 Avoir. Pound	=.4536 Kilo.
1 Ton	=.907 Tonneau.

CHAPTER III.

SUBTRACTION OF ALL KINDS OF NUMBERS.

Section 1. SUBTRACTION OF WHOLE NUMBERS.

Subtraction is finding the difference between two or more than two numbers. A short dash is the sign of subtraction when used between two numbers; thus, $4-2=2$. If two numbers are equal the difference is nothing or a cipher, 0; thus $6-6=0$. Or, the difference of two numbers is the result of taking a less number from a greater. The less number is called the subtrahend and the greater the minuend. If the remainder is added to the subtrahend, the result equals the minuend.

1ST PRINCIPLE. — All numbers must be of the same name before they can be actually subtracted.

Various methods of performing subtraction may be adopted in practice.

1st. *Mental Operations*. Thus, $20-2=18$; $25-2=23$; $50-10=40$. Similar exercises can be given to any extent, to secure accuracy and rapidity, until the pupils can readily take any of the nine digits from any number larger than itself.

The circular diagram, shown on page 22, may be used effectually in teaching a class to be accurate and quick in subtracting. When it is used, the pupils must keep in mind that they are to find the difference between the centre figure and any one in the circumference. To make the pupils more efficient, the figures may be changed, and it is well to require the numbers to be added as well as subtracted.

The numeral frame may be used also, with advantage, to give a correct idea of the *meaning* of subtraction.

In writing the minuend and subtrahend for subtraction, usually the smaller is written directly under the larger; and, *always*, those figures of the same name must be in the same column.

1. Find the difference between 87 and 66.

1.	**2.**	**3.**	
Thus, 8 7 –	or, 8 7 6 –	or, 9 8 7 5 –	= Minuend.
6 6	6 6 5	4 6 5 3	= Subtrahend.
2 1	2 1 1	5 2 2 2	= Remainder.

4. From 8,745 take 7,456. Thus,

$$\begin{array}{r} 8745- \\ 7456= \\ \hline 1289 \end{array}$$

In this example six units cannot be taken from five units; but we can take one ten from the preceding four tens, making ten units to be added to the five units, equaling fifteen units; and then we can take six units from fifteen units, equaling 9 units, for the unit figure in the remainder.

Now, as we took one ten from the four tens, only three tens remain, from which we are to take five tens; which cannot be done until we take one hundred, or ten tens, from seven hundred; adding this to the three tens we get thirteen tens; from this now take five tens, and eight tens are left for the tens' place. Now we have taken one hundred from the seven hundred, leaving six hundred, from which we can take the four hundreds, leaving two hundreds for the hundreds' place. Now we have only to take seven thousands from eight thousands, leaving one thousand for the thousands' place, making the whole remainder 1,289. This example and the explanation illustrate every case of difficulty in actual subtraction. The following examples will furnish additional practice and illustration: —

5.

From 76423 −
57254

6.

From 634261 −
455173

7.

From 5000 −
3452

8.

From 600201 −
524680

9. How many years from the date of the Declaration of Independence, July 4, 1776, to the 4th of July, 1885?

Section 2. SUBTRACTION OF DECIMALS.

Decimal numbers are written and subtracted like whole numbers. Numbers of the same name must always be written under each other; that is, tenths under tenths, hundredths under hundredths, thousandths under thousandths, etc. Remember that one unit is ten times as much as one tenth, and one tenth is ten times one hundredth, and one hundredth is ten times one thousandth, etc.

10.

	Tenths.	Hun.	Thou.	10-th.
From	.5	4	2	3 −
	.1	3	1	2

11.

From 2.658234 −
1.547543

12.

From 25.0025 −
14.13425

13.

From 10.000005 −
6.100001

14. From twenty-one, and three hundred and twenty-five hundred-thousandths, take fifteen hundred and twenty-five millionths.

15. What is the difference between one million and one millionth? Between five thousand and five thousandths? Between one million and five thousandths?

Note.—The teacher should suggest and supplement as many examples as may be necessary to make this subject thoroughly understood. Take nothing for granted.

Section 3. SUBTRACTION OF COMMON FRACTIONS.

To subtract common fractions they must in the first place be of the *same denomination;* in the second place, they must be *simple* fractions; and, in the third place, they must be of the *same name.*

Again, all *compound* and *complex* fractions must be reduced to simple fractions. (See pages 31, 32). Finally, the numerator of the subtrahend must be taken from the numerator of the minuend, and the remainder placed over the common denominator.

16. From $\frac{3}{4}$ take $\frac{3}{6}$, expressed thus, $\frac{3}{4}-\frac{3}{6}$. First, find by inspection the least common multiple of the denominators, to get the fractions to the same name. This we find to be 12, *the least number* of which we can get an *even fourth* and *sixth.* As we have before seen, the new numerators will be $\frac{3}{4}\times 12=9$, and $\frac{3}{6}\times 12=6$. The fractions then are $\frac{9}{12}-\frac{6}{12}=\frac{3}{12}=\frac{1}{4}$.

17. From $\frac{6}{7}$ take $\frac{5}{14}$.

We see that the smallest common denominator is 14. We have $\frac{12}{14}-\frac{5}{14}=\frac{7}{14}=\frac{1}{2}$.

18. What is the difference between 4 and $2\frac{3}{8}$?

$$\begin{array}{r} 4 \\ 2\frac{3}{8} \\ \hline 1\frac{5}{8} \end{array}$$

OPERATION: Here we take one unit ($\frac{8}{8}$) from 4, (leaving 3), from which $\frac{3}{8}$ is to be subtracted; then taking 2 from 3 we get $1\frac{5}{8}$ for the full remainder. This course can be taken often in subtracting a mixed number from a whole number.

19. From $7\frac{6}{7}$ take $5\frac{1}{7}$. Here subtract without change.

20. From $\frac{5}{7}$ take $\frac{1}{3}$. $\frac{15}{21}-\frac{7}{21}=\frac{8}{21}$.

21. Mr. James sold $\frac{1}{3}$ of his farm to one man, $\frac{1}{4}$ to another, and $\frac{1}{8}$ to another. How much was left?

The least common name for the fractions is 24ths. The whole farm is $\frac{24}{24}$. The fraction will be found by first adding the sales; second, getting the fractions to the same name, *twenty-fourths;* thus, $\frac{8}{24}+\frac{6}{24}+\frac{3}{24}=\frac{17}{24}$, taken from $\frac{24}{24}=\frac{7}{24}$.

22. The sum of two numbers is $124\frac{1}{4}$, and the less is $36\frac{1}{5}$; what is the greater?

23. What fraction added to the sum of $\frac{1}{8}+\frac{5}{12}+\frac{5}{18}$ will make $\frac{133}{144}$?

24. A man bought a ton of hay for $\$15\frac{3}{8}$, a barrel of flour for $\$9\frac{5}{12}$, a barrel of apples for $\$3\frac{7}{16}$. What change must he receive in payment if he pays three ten-dollar bills?

Section 4. SUBTRACTION OF DENOMINATE NUMBERS.

As the learner has already been taught to read denominate numbers, and to add them, no directions are needed here except the following: —

Write the terms of the subtrahend, in the order of their value, under the corresponding terms of the minuend, and then begin with the terms of the lowest value and subtract as in whole numbers, bearing in mind always how many parts of each name make a unit of the next higher name.

1. From £15, 12s., 6d., take £4, 6s., 3d.

2. From £24, 8s., 4d., 3far., take £15, 10s., 8d., 2far.

3. From 6yrs., 8mo., 24da., 12h., 45mi., 24sec., take 3yrs., 6mo., 18da., 8h., 45m., 30sec.

Note. — Usually 12 months are considered a year, and 30 days a month; but if exact time is required, the exact number of days in each month must be reckoned.

4. How long has it been since the Declaration of Independence, July 4, 1776, to the present date?

5. From 25rds., 2yds., 2ft., 6.3in., take 12rds., 4yds., 11.6in.

6. If from a hogshead of molasses 14gal., 1qt., 1pt. be drawn at one time, 10gal., 3qt. at another time, and 29gal., 1pt. at another, how much will remain?

Note. — Sometimes denominate fractions are to be added and subtracted; in such cases the value of each fraction must be found in whole numbers.

7. From £$\frac{1}{4}$ take $\frac{5}{6}$s.

OPERATION: $\frac{1}{4} \times 20s. = 5s. = 60d.$; and $\frac{5}{6} \times 12d. = 10d.$ $60d. - 10d. = 50d. = 4s., 2d.$

Other examples may be given by the teacher.

CHAPTER IV.

MULTIPLICATION.

Section 1. MULTIPLICATION OF WHOLE NUMBERS.

Of course, nothing can be *well done* in the operation of multiplying, without a perfect command of the multiplication table.

The teacher cannot be too particular in requiring a perfect command of all the tables herein given.

But when large numbers are to be multiplied, or repeated by multipliers having single or combined digits, it is generally best to write the larger number *first*, whether it is the multiplier or multiplicand; and the smaller number under it, with units of the same order under each other. *But the pupil must always keep in mind that the number to be repeated is the multiplicand, and that the number which indicates the number of repetitions is the multiplier.*

OPERATION: 1st. Take the unit figure of the lowest order of the multiplier, and by it multiply each figure in the multiplicand, beginning with the lowest order, remembering that every ten units of any order make *one* unit in the next higher order.

2d. Take the multiplying unit of the next higher order, and by it multiply the multiplicand again, as before, putting the *first* product under the multiplying unit, and so on, until each figure of each order in the multiplier has been used. If the figure of any order in the multiplier is a cipher, use the next significant figure, putting the first product under the multiplying figure.

3d. Finally, add the several partial products, and the sum will be the product sought.

1. Multiply 75 by 25. 75 × 25.

7 5 ×	Mu'nd.
2 5	Mu'er.
3 7 5	Par. Pr.
1 5 0	Par. Pr.
1 8 7 5	Whole Pr.

ILLUSTRATION: Here the 5 units of the multiplicand multiplied by the 5 units of the multiplier make 25 units, which are equal to 2 tens and 5 units. The 5 units are put under the multiplying unit, as the unit figure of the first partial product. Keep the 2 tens in mind, and add them to the product of the 7 tens multiplied by the 5 units, which equals 35+2=37 tens, or 3 hundreds and 7 tens. As there is no other figure in the multiplicand, write the 3 hundreds and 7 tens in their proper order Next, multiply the 5 units of the multiplicand by the 2 tens of the multiplier, making 10 tens, or one hundred and no tens. Write the 0 tens in the tens' place. Keep the one hundred in mind, and add it to the product of the 7 tens of the multiplicand by the 2 tens of the multiplier, making 14 tens and 1 ten=14+1=15 tens=1 thousand and 5 tens, to be written in their proper order. Now add the partial products, 375+1,500=1,875.

This operation explains all the operations necessary in multiplication, and should be thoroughly understood.

2. Multiply 756 by 352. 756 × 352; or as usual,

7 5 6 ×				
3 5 2				
1 5 1 2	The first partial product is		1512	units;
3 7 8 0	the second partial product is		3780	tens,
2 2 6 8	and the third ,,	,, ,,	2268	hund's.
2 6 6 1 1 2			266112	

If to any figure, or number, a cipher is annexed, that figure or number is multiplied by ten; if two ciphers are annexed it is multiplied by one hundred, and if three ciphers are annexed, by one thousand, and so on. Again, if one or more ciphers are annexed to both the multiplier and the multiplicand, the significant figures must first be multiplied, and as many ciphers must be annexed to the product as are annexed to both factors.

3. Multiply 2,500 by 50.

$$\begin{array}{r} 2500 \\ 50 \\ \hline 125000 \end{array}$$

4. Multiply 4,682 by 3,500.

5. Multiply 6,780 by 5,000.

6. Multiply 678,256 by 34,583.

7. How many seconds in one day?

8. How many seconds old are you?

9. A farmer sold 468lbs. of pork at 6cts. a pound, and 48lbs. cheese at 7cts. a pound; and received in payment 42lbs. sugar at 9cts. a pound, 100lbs. nails at 6cts. a pound, 108yds. of sheeting at 10cts. a yard, and 12lbs. of tea at 95cts. a pound. How does the farmer's account stand?

Note. — The teacher must improvise and supplement as many examples as may be necessary to make the pupils accurate and rapid in their operations.

Section 2. MULTIPLICATION OF DECIMAL NUMBERS.

1. If a number containing a decimal expression is to be multiplied by a whole number, the product will have just as many decimal places as are in the multiplicand.

2. If a whole number is to be multiplied by a decimal number, the product will have just as many decimal places as are in the multiplier.

3. If both factors have decimal places, the product will have as many decimal places as are contained in both factors.

EXPLANATION. — As multiplying ten by ten produces one hundred, so multiplying tenths by tenths produces hundredths; as multiplying a hundred by ten produces a thousand, so multiplying hundredths by tenths produces thousandths, etc. etc.

4. If the product of the significant figures of a decimal expression does not contain as many figures as there are decimal places in the factors, then as many ciphers should be prefixed to the product as will make its decimal places equal to the sum of those in the two factors. Thus, $.06 \times .06 = .0036 = \frac{6}{100} \times \frac{6}{100} = \frac{36}{10000}$.

1. Multiply .5 by .5. $\frac{5}{10} \times \frac{5}{10} = \frac{25}{100} = .25$.

The art of writing decimal fractions consists in expressing their value, like whole numbers, without expressing the denominators. (See Chap. I, page 10.)

2. Multiply .25 by 8. $.25 \times 8 = 200$ hundredths, or $\frac{200}{100} = 2$, or 2.00.

3. Multiply 87.5 by 12. $87.5 \times 12 = 1,050.0$.

4. Multiply 125.275 by 225.

5. Multiply .1275 by 1275.

6. Multiply 6.00625 by 65.

7. Multiply .000525 by 525.

8. Multiply 625. by .5.

9. Multiply 1275 by .1275.

10. Multiply 525 by .000525.

11. Multiply 12,340 by .25.

12. Multiply 87.25 by 3.625.

13. If an acre of land produces 127.25 bushels of potatoes, how many bushels will 4.375 acres produce?

14. Multiply 84 tenths by 244 hundredths.

15. What will 204.7 acres cost at $97.75 per acre?

16. What is the cost of 3.625 bales of cloth, each bale containing 36.75 yards, at $0.85 a yard?

17. Multiply .0062 by .0008.

18. Multiply five hundredths by five thousandths.

19. If a man receive $0.75 a day, how much will he receive in four weeks, Sundays excepted?

Section 3. MULTIPLICATION OF COMMON FRACTIONS.

A fraction can be multiplied in two ways: —

1st. By repeating the number of parts taken, that is, by multiplying the numerator.

2d. By increasing the size or value of the parts, or by dividing the denominator.

But if the numerator and denominator be each multiplied or divided by the same number, the value of the fraction remains unchanged.

Reason. — Multiplying the numerator increases the number of parts taken a certain number of times; and multiplying the denominator by the same number decreases the size or value of the parts the same number of times.

Hence, to multiply a fraction by any number, divide the denominator, if it can be done without a remainder; otherwise, multiply the numerator.

1. Multiply $\frac{1}{3}$ by 3. Thus, $\frac{1\times 3}{3}=\frac{3}{3}=1$; or $\frac{1}{3\div 3}=\frac{1}{1}=1$.

2. Multiply $\frac{5}{6}$ by 6. Thus, $\frac{5\times 6}{6}=\frac{30}{6}=5$; or $\frac{5}{6\div 6}=\frac{5}{1}=5$.

3. Multiply $\frac{10}{12}$ by 4. Thus, $\frac{10\times 4}{12}=\frac{40}{12}=3\frac{4}{12}$; or $\frac{10}{12\div 4}=\frac{10}{3}=3\frac{1}{3}$.

4. Multiply $4\frac{1}{3}$ by 3. Thus, $4\frac{1}{3}\times 3=12\frac{3}{3}=12+1=13$; or, reduce $4\frac{1}{3}$ to thirds $=\frac{13}{3}\times 3=\frac{39}{3}=13$; or $\frac{13}{3\div 3}=\frac{13}{1}=13$.

5. Multiply $16\frac{2}{3}$ by 6. Thus, $16\frac{2}{3}\times 6=96\frac{12}{3}=96+4=100$; or, reduce $16\frac{2}{3}$ to thirds; then $\frac{50\times 6}{3}=\frac{300}{3}=100$.

6. Multiply $66\frac{2}{3}$ by 12, and add to the product $33\frac{1}{3}\times 6$.

7. What will $14\frac{3}{4}$ bushels of apples cost at $3 per bushel?

8. What will $\frac{11}{15}$ of a ton of butter cost at a rate of $225 per ton?

Note 1. — A whole number may be multiplied by a fraction by taking such a part of the whole number as the fraction is a part of unity; or, multiply by the numerator and divide by the denominator.

2. Again, a *fraction* may be multiplied by a fraction in the same way, by taking such a part of the fractional multiplicand as the multiplying fraction is a part of unity.

9. (Under *Note 1.*) $6 \times \frac{1}{3}$. Here $\frac{1}{3}$ is a third part of unity; so $\frac{1}{3}$ of $6 = 2 =$ or $6 \times \frac{1}{3} = \frac{6}{3} = 2$.

10. Multiply 7 by $\frac{1}{3}$. We cannot get an even third of 7, so we may reduce 7 to thirds$= \frac{21}{3}$. Now $\frac{1}{3} \times \frac{21}{3} = \frac{7}{3} = 2\frac{1}{3}$.

11. Multiply 81 by $\frac{7}{9}$. Thus, $\frac{1}{9} \times 81 = 9$, and $\frac{7}{9}$ of $81 = 7 \times 9 = 63$.

12. Multiply 144 by $\frac{15}{16}$.

13. (Under *Note 2.*) Multiply $\frac{3}{4}$ by $\frac{5}{6} = \frac{3}{4} \times \frac{5}{6}$, a compound fraction. The multiplicand is $\frac{3}{4}$, and we want to get five sixths ($\frac{5}{6}$) of it. (To get a sixth of any number, divide the number into six times as many parts and take one part.) Thus, $\frac{3}{4}$ changed to six times as many parts becomes $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$. Now $\frac{1}{6} \times \frac{18}{24}$ is $\frac{3}{24}$ and $\frac{5}{6} \times \frac{18}{24} = \frac{15}{24}$. The result gives us the rule for multiplying one fraction by another.

RULE. — *Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

A compound fraction may have any number of simple fractions to be multiplied together; thus: —

14. Find the product of $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$. We can simplify this compound fraction first, as follows: $1 \times 2 \times 3 \times 4 = 24$ for a numerator; then $2 \times 3 \times 4 \times 5 = 120$ for a denominator, which will make $\frac{24}{120} = \frac{1}{5}$. But in using common fractions the numerator is always considered a *dividend* and the denominator a *divisor*.

In the above example each numerator is a dividend and each denominator a divisor. We have then $\frac{2}{2} = 1$, $\frac{3}{3} = 1$, and $\frac{4}{4} = 1$. The fractions will be then $\frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{5} = \frac{1}{5}$; or the example may be simplified by canceling equal factors in the numerator and denominator; thus, $\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{1}{5}$.

15. Multiply $\frac{3}{5}$ by $\frac{5}{9}$ by $\frac{5}{6}$. The 5 in the denominator will cancel one 5 in the numerator, and the 3 in the numerator being a factor of 9 in the denominator will make the 9 a 3; thus, $\frac{\cancel{3}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{9}_3} \times \frac{5}{6} = \frac{5}{3 \times 6} = \frac{5}{18}$.

16. Multiply $2\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}$. Here $2\frac{1}{2}$ is a mixed number and must be reduced to halves; thus, $\frac{5}{2} \times \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{5}{8}$.

17. Multiply $5\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{4} \times \frac{1}{6}$.

18. Multiply $6\frac{1}{4} \times 3\frac{1}{3} \times \frac{2}{3} \times \frac{7}{8}$.

19. At \$$2\frac{1}{4}$ per yard, what cost $6\frac{5}{8}$ yards?

20. A owned $\frac{5}{9}$ of a ticket, B owned $\frac{1}{3}$, and C $\frac{1}{9}$. The ticket was worth \$2,700. What was the share of each?

21. What is the product of $\frac{1}{9} \times 8 \times \frac{2}{3} \times 7 \times \frac{3}{8} \times 9$?

Section 4. MULTIPLICATION OF DENOMINATE NUMBERS.

In this operation the multiplier is always an *abstract number*, simply representing a certain number of units; and the product will always be of the same name as the multiplicand.

DIRECTIONS: 1. Write the numbers in the order of their value.

2. Multiply as in whole numbers, remembering the *name* and the *tables* of parts required to make a higher unit.

1. Multiply £2, 5s., 6d., 2far., by 6. (Table of English Money.)

£	s.	d.	far.
2	5	6	2 ×
			6
13	13	3	0

OPERATION: 2far. × 6 = 12far. 12far. ÷ 4far. = 3d. and no farthings. Write a cipher in farthings' place and add 3d. to the next product. 6d. × 6 = 36d. 36d. + 3d. = 39d. 39d. ÷ 12d. = 3s. and 3d. Write 3d. in the place of pence, and add the 3s. to the next product. 5s. × 6 = 30s. 30s. + 3s. = 33s. 33s. ÷ 20s. = £1 and 13s. Write the 13s. in the place of shillings, and add the £1 to the next product. £2 × 6 = £12. £12 + £1 = £13. Write the £13 in the place of pounds.

The above illustration is sufficient for all kinds of denominate numbers.

2. Multiply 2bu., 3pk., 6qt., by 9. (See Table of Dry Measure.)

3. Multiply 8gal., 3qt., 1pt., 3.25gi., by 96.

Note. — If the multiplier is a large, composite number (that is, made up of two or more factors), the multiplicand may be multiplied by one of the factors and its product by the other factor. In the last example, 96 = 8 × 12, to be used as follows: —

gal.	qt.	pt.	gi.	
8	3	1	3.25 ×	
			8	
71	3	0	2 ×	
			12	
861	3	0	0	= 13 hhd., 42 gal., 3 qt.

4. How many bushels of grain in 47 bags, if each bag contains 2bu., 2pk., 6qt.?

5. If 1 barrel of apples contains 2bu., 3pk., 1qt., how many bushels are there in 56 barrels?

6. A farmer sold 4 loads of oats, each load averaging 44bu., 3pk., at $0.75 a bushel. What did he receive for the whole? Ans. $134.25.

7. Multiply 7T., 15cwt., 10.5lbs., by 1.7.

Numerous other examples should be given by the teacher.

CHAPTER V.

DIVISION.

Section 1. DIVISION OF WHOLE NUMBERS.

Division consists in finding how many times one number is contained in another; or how many times a certain number can be taken from another.

Before the operation can be performed, the terms must be of the same name. The result, or quotient, will generally have the same name as the dividend.

1. *Mental Operations.* — Take the multiplication table and give the pupil the product and one factor, to find the other. The pupil must be thorough and quick. Take, for instance, 12, and the factors, 2 or 3 or 4 or 6; thus $12 \div 2 = 6$; $12 \div 3 = 4$; $12 \div 4 = 3$; $12 \div 6 = 2$.

Then take 56, and the factors 7 and 8. Follow up this exercise as long as necessary. Every possible means should be used to familiarize pupils with the tables.

If the divisor consist of only one order of units, the process of division is simple and plain, thus: —

1. Divide 444 by 4. Thus $\frac{4)444}{111}$. Here the divisor, 4 units, is contained in four units of hundreds, one hundred times; and in four units of tens, one ten, or ten times; and in four units, one time, or once; making the quotient one hundred and eleven.

2. Divide 712 by 8. Here the 8 units will not measure the 7 hundred any hundred times, so they must be reduced to the next lower order, tens, making 7 tens, or 70; then add the one ten, making 71 tens. Now 8 units are contained in

71 tens, 8 tens times, with 7 tens yet undivided. Reduce the 7 tens to units, the next lower name, making 70 units; to which add the 2 units, and we have 72 units, which contain 8 units 9 units times. This makes the quotient 89.

3. Divide 6,895 by 5; also 8,928 by 8; also 1,728 by 12.

4. Divide 225 by 15.

5. Divide 14,625 by 45.

6. Divide 78,125 by 125.

7. Divide 326,482 by 24.

8. Divide 1,500 by 50.

Note.—When the divisor has one or more ciphers on its right, cut them off, and also as many figures from the right of the dividend. Divide by the significant figures, and prefix the remainder, if any, to the ciphers cut off.

9. The annual receipts of a company are $570,685; what is the average for each of 313 working-days?

10. Divide 6,228 by 36; resolving 36 into two factors.

11. Divide 757 by 30, resolved into factors 5 × 6, by short division; thus:—

$$\begin{array}{r|l} 5 & 757 \\ \hline 6 & 151\frac{2}{5} \\ \hline & 25\frac{7}{30} \end{array}$$

The first division produces a quotient of $151\frac{2}{5}$, and the second division produces a remainder of $1\frac{2}{5}$; simplified and divided by 6 it becomes $\frac{7}{30}$.

12. Divide 73,522 by 135, resolved into factors, 3 × 5 × 9.

13. A man pays out every week of six days, to his employees, $1,012.50, at $2.25 a day. How many men does he employ?

Section 2. DIVISION OF DECIMALS.

1. Write the terms as in whole numbers.

2. Before attempting to divide, make the number of decimal places in the *divisor* and *dividend equal*, when the quotient will be a *whole number*, if the dividend is greater than the divisor; but if not greater, or if there is an undivided remainder, annex one cipher to the dividend, and divide for *tenths*, and two ciphers for hundredths, and so on.

The above directions are universal in their application; they insure *rapidity* and *accuracy*, and overcome all the difficulties so common in pointing off decimals.

1. Divide 15 by 5 tenths. 15=150 tenths; and 5 tenths is contained in 150 tenths, 30 times. $\frac{150}{5}=30$.

2. Divide 25 by 25 millionths. 25 (whole number) is equal to 25.000000; divided by 25 millionths, the quotient is 1,000,000. Or, (1) Write the divisor and dividend as in whole numbers; thus, .000025)25. (2) Annex six ciphers to 25 and it becomes 25.000000 for the real dividend.

3. Divide 375 by .375; thus, .375)375.000=1,000.

4. Divide .6 by 15; thus, 15.0).6=0 units, 0 tenths, and 6 hundredths; or, 15.0).600=0.04.

5. Divide .375 by 375.

6. Divide .675 by 135.

7. Divide .6 by .3=2.

8. Divide .6 by .03.

9. Divide 0.05 by .25.

10. Divide $80 by $0.25.

11. Divide 32.8125 by $6.56\frac{1}{4}=32.8125 \div 6.5625$.

12. At \$287$\frac{2}{5}$ for one horse, how many horses can be bought for \$7,185?

Section 3. DIVISION OF COMMON FRACTIONS.

A fraction can be divided in two ways: —

1st. By dividing the numerator; thereby diminishing the number of parts taken, while the size or value of the parts remains the same.

2d. By multiplying the denominator; thereby diminishing the size or value of the parts, while the number of parts remains the same.

But if the numerator and denominator be each divided by the same number, the value of the fraction remains the same, because the value of the parts is increased in the same ratio that the number of parts is diminished.

Again, a whole number may be divided by a fraction, by first reducing the whole number to an improper fraction, or to parts of the same name as the dividing fraction, and then dividing the numerator of the dividend by the numerator of the divisor.

Again, if one fraction is to be divided by another fraction, both fractions must *first* be of the same denomination; and *second*, they must be reduced to equivalent fractions having the same denominator, by inspection or otherwise, as heretofore shown, then divide, as in the last case, one numerator by the other.

It may sometimes be desirable, and perhaps necessary, to find the Greatest Common Divisor (G. C. D.) of two or more numbers.

RULE. — *Divide the greater of two numbers by the less; and then divide the last divisor by the last remainder, and so continue to do until the remainder will exactly divide the last divisor. This last remainder is the G. C. D.*

If more than two numbers are used, first find the G. C. D. of two; and then of their G. C. D. and any other number, and so on.

Note.—In practice, this rule will rarely be required, as common divisors can generally be found by inspection. But the principles, stated above, must be illustrated and made clear to the pupils, by oral and blackboard instruction.

1. Divide $\frac{6}{7}$ by 3. Thus, $\frac{6 \div 3}{7} = \frac{2}{7}$, or $\frac{1}{3} \times \frac{6}{7} = \frac{2}{7}$, or $\frac{6}{7 \times 3} = \frac{6}{21} = \frac{2}{7}$.

2. Divide $\frac{7}{8}$ by 3. Thus, $\frac{7}{8 \times 3} = \frac{7}{24}$. Here we diminish the size of the parts while the number remains the same.

3. Divide 12 by $\frac{3}{4}$. Here reduce 12 to fourths $= \frac{48}{4}$, the same name as the divisor; then $\frac{48}{3} = 16 =$ Ans.

Note.—When fractions of the same denomination have the same name, they can be divided, one by the other, by dividing the numerator of one by the numerator of the other.

4. Divide 11 by $5\frac{1}{2}$. Here make a simple fraction of $5\frac{1}{2} = \frac{11}{2}$; and then reduce 11 to halves $= \frac{22}{2}$, then $\frac{22}{11} = 2 =$ Ans.

5. How many yards of cloth can be bought for $75, if one yard costs $2.50? Ans.—Either $\frac{75.00}{2.50}$ or $\frac{75}{1} \div \frac{5}{2}$ or $\frac{150}{5} = 30$.

6. Divide $\frac{3}{4}$ by $\frac{1}{3}$. We see by inspection that $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{3} = \frac{4}{12}$; then the numerator $9 \div 4 = \frac{9}{4} = 2\frac{1}{4} =$ Ans.

Note.—One fraction can be divided by another fraction by inverting the divisor and then proceeding as in multiplication of fractions.

7. Divide $\frac{7}{8}$ by $\frac{2}{3}$. Here the least common name is 24: and $\frac{7}{8}$ of $24=21$, and $\frac{2}{3}\times 24=16$. Then, $\frac{21}{16}=1\frac{5}{16}=$ Ans.

8. Divide $\frac{4}{7}$ by $\frac{2}{9}$, or $\frac{4}{7}\div\frac{2}{9}=\frac{36}{14}=2\frac{8}{14}$.

9. Divide $\frac{2}{3}\times\frac{3}{4}$ by $\frac{7}{8}\times\frac{4}{7}$. Simplify, and proceed as before.

10. Divide $\frac{3}{4}$ by $4\frac{1}{2}$. Make a simple fraction of $4\frac{1}{2}=\frac{9}{2}$ and then proceed as before.

11. Divide $7\frac{5}{6}$ by $3\frac{1}{4}$. $7\frac{5}{6}=\frac{47}{6}$ and $3\frac{1}{4}=\frac{13}{4}$, then $\frac{47}{6}\div\frac{13}{4}$.

12. Divide $\frac{3}{4}$ by $\dfrac{\frac{1}{2}\times\frac{2}{3}}{4\frac{1}{2}}$. Simplify the divisor $=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$; then $\frac{1}{3}\div 4\frac{1}{2}$, or $\frac{1}{3}\div\frac{9}{2}=\frac{2}{6}\div\frac{27}{6}=\frac{2}{27}$. Now, $\frac{3}{4}\div\frac{2}{27}=81\div 8=10\frac{1}{8}=$ Ans.

13. Divide $\dfrac{\frac{2}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{3}{4}}$ by $\frac{4}{5}$.

14. Divide $\dfrac{5\frac{1}{2}}{\frac{1}{3}\times\frac{1}{5}}$ by $\dfrac{\frac{2}{3}+\frac{1}{4}}{6\frac{1}{4}}$.

Note. — The above examples embrace about every variety of fractional numbers coming under division; but the teacher should give extra examples, varied, of course, under each of the model examples, after making clear every process.

Section 4. DIVISION OF DENOMINATE NUMBERS.

Denominate numbers may be divided like whole numbers, after reducing all the terms to the same denomination; and after division the quotient may be reduced back to such higher denominations as may be necessary.

But the usual way is *first* to divide the highest denomination; and if there is any remainder reduce it to the next lower denomination, and add to it the term in the dividend of the same kind; and then divide the sum, placing the quotient under the term of the same name; and if there is still a remainder, reduce, add, and divide as before, placing the quotients in their order, until all the terms are divided.

This rule applies to the division of all kinds of numbers.

1. Divide £25, 18s., 6d., equally between 6 persons. Thus, £25, 18s., 6d. ÷ 6. First, £25 ÷ 6 = £4, and one pound, or 20s., undivided. Add 20s. to 18s. = 38s. 38s. ÷ 6 = 6s., and 2s. or 24d. undivided. Add 24d. to 6d. = 30d. 30d. ÷ 6 = 5d. exactly. The several quotients, £4, 6s., 5d., make the required quotient.

2. Divide 25yd., 4ft., 3½in. by 21 and 42, or by the factors 3×7 and 7×6.

3. Divide 282bu., 3pk., 1qt., 1pt., by 9, 10, 12.

4. Divide 336bu., 3pk., 4qt., by 4bu., 3pk., 2qt.

5. Divide 5T., 15cwt., 3qr., 20lbs., by 56½. Or, a man received a cargo of corn weighing 5T., 15cwt., 3qr., 20lbs. How many bushels were there, if one bushel weigh 56½lbs?

6. If a town 4 miles square be divided into 64 farms of equal size, how many acres will there be in each farm?

Note.—To insure uniformity in the methods of *writing* and *reading* numbers, connected by the signs $+$, $-$, $\times$, and $\div$, the following directions are offered, which should be made familiar to all pupils.

1. Let the operations indicated by the signs be performed in their order, from the first left-hand sign.

2. If the sum, difference, product, or quotient of two or more numbers is to be added, subtracted, multiplied, or divided by any number, said sum, difference, etc., should each be joined by a vinculum, or parenthesis.

Thus the first example, $12+6-2\times2-3\times2=58$; to be read, twelve added to six; the sum diminished by two; the remainder multiplied by two; the product diminished by three, and the remainder multiplied by two, equals fifty-eight.

The second example, $12+6-(2\times 2)-(3\times 2)=8$, is to be read, twelve added to six, diminished by twice two, and the remainder diminished by three times two, equals eight.

CHAPTER VI.

PRACTICAL AND TEST EXERCISES.

The following exercises are designed to furnish the *practical training* needed for discipline, and for fixing in the mind the foregoing fundamental principles.

Section 1. PER CENT. AND PERCENTAGE.

1. Per cent. means by the hundred; or, taking a certain number of parts of a thing divided, or supposed to be divided, into a hundred equal parts. Thus, 5 per cent. (or 5%) means five parts of a thing, or of a number, divided into a hundred equal parts.

Six per cent. is $\frac{6}{100}$, or .06, of anything. Fifty per cent. is $\frac{50}{100}=\frac{1}{2}$ of anything, and one hundred per cent. is $\frac{100}{100}$ of a thing, equal to the number itself.

In business transactions, in order to provide a convenient and uniform mode of making arithmetical comparisons, things and numbers are frequently supposed to be divided into one hundred equal parts.

2. Percentage is the result, or what is obtained by finding the *per cent.* of a number or quantity. The *percentage* of $100 at 6% is $6; of $200, it is $12; and of $250, it is $\$6\times2\frac{1}{2}=\$15$.

The most frequent use of percentage is in connection with money transactions, such as loans, credits, notes, etc.

Take special notice that if $100 is loaned at 6%, it is a transaction of the same nature as that of selling 100 pounds of sugar at 6 cents a pound; in other words, it may be called selling money for a certain number of parts out of every hundred, for the use of it for a fixed time. Hence, to find the percentage, or the interest of any sum of money, for a specified time: ***Multiply the given sum by the per cent. required, and point off the product as in multiplying decimals.***

Remember that *per cent.* means by the hundred, and hence percentage always occupies the hundredths' place; so that if there are only *dollars* loaned, the product will be cents; but if there are dollars and cents in the loan, the product will have four decimal places.

1. What is the interest of $155 at 6%, for one year?

OPERATION: $\$155 \times .06 = \$9.30$.

2. What is the interest of $175.75 at 6%, for one year?

OPERATION:

$$\begin{array}{r} \$175.75 \times \\ .06 \\ \hline \$10.5450 \end{array} = \$10.54\tfrac{1}{2}$$

Note. — If the loan runs for years or parts of a year, multiply the interest for one period, or year, by the number of periods, or of years, and by the fractional part of a year, if any.

3. What is the percentage, or interest, of $75 at 6%, for one, two, and $3\frac{1}{2}$ years?

4. What is the interest of $345.75 at 7%, for two years?

OPERATION: $\$345.75 \times .07 \times 2 = \$48.405$.

Note. — If money is loaned for years, months, and days, at a certain per cent. per annum (or by the year), we must,

First, find the exact time in years, months, and days, as taught under subtraction of denominate numbers; and

Second, find the percentage for *one year*, and then for the number of years, according to directions given before. If there are months, take $\frac{1}{12}$ of the interest for 12 months, as the interest for *one* month, and multiply it by the number of months; and if there are days, take $\frac{1}{30}$ of the interest for 1 month, or 30 days, and multiply it by the number of days. The sum of the percentages thus found is the whole interest sought.

Or, if the *exact* interest is required, *first* find the interest for the year or years, as before; and then multiply the interest for one year by the exact number of days, and divide the product by 365, the number of days in a year, and then add the sums for the required interest.

5. What is the interest of $175 from January 1, 1882, to November 30, 1883, at 6% per annum?

	yr.	mo.	da.
1st. Find the time; thus,	1883	11	30
	1882	1	1
	1	10	29

2d. Find the interest for 1yr., 10mo., 29da., as above directed. Ans. $20.095.

Operation for exact interest: $\$175 \times .06 = \$10.50$. $\$10.50 \times \frac{333}{365} = \$9.58$. $\$9.58 + 10.50 = \$20.08$.

Note. — If the time is in years, multiply the principal by the rate per cent. and number of years, for interest.

The following is a good general rule for finding the interest, when there are months and days.

1. *Find the exact time.*

2. *Find the interest on one dollar for the given rate and time, at 6%, as follows. Interest for one year .06; for each month .005 (or 5 mills); for every six days, .001 (or 1*

mill); and for any number of days less than six, the proportional part of a mill.

3. *Add the several results, and multiply the sum by the principal. The product will be the interest sought.*

4. *If the rate is any other than 6%, add to, or subtract from, the 6% such a part of itself as the required per cent. exceeds, or falls short of, 6%.*

ILLUSTRATION: If the time is 3yrs., 8mos., 18 da., at 6%, we find 3yrs. at 6% is \$0.18; 8mos. will be $\frac{8}{2}$ cents, or .04, and 18da. will be $\frac{18}{6}$ mills=\$0.003. The sum of the results will be \$0.18+\$0.04+\$0.003=\$0.223, which must be multiplied by the principal.

AMOUNT.—To find the *amount* of any sum of money on interest at a given rate and time, multiply the amount of one dollar for the rate and time, by the principal, and the product will be the required amount.

6. What is the interest of \$525.50 from July 4, 1876, to December 31, 1880, at 8%? ANS. \$188.8296+.

7. What will be the *amount* of \$525 on interest at 6%, for 2 years, 6 months?

OPERATION: The interest on one dollar for the time at 6% is \$0.15, and \$0.15+\$1=\$1.15=amount of one dollar for $2\frac{1}{2}$ years. \$1.15×\$525=\$603.75=ANS.

Sometimes we may want to find the *rate* per cent. when the principal, interest or percentage, and time are given.

RULE.—*Divide the whole percentage or interest by the interest of the principal for the time given at 1%.*

8. The principal, or base, is \$96; the interest for 5 years is \$28.80; what is the rate per cent.?

OPERATION: The interest of \$96 at 1% is \$0.96 for 1 year, and for 5 years=\$0.96×5=\$4.80. \$28.80÷4.80=.06, or 6%.

Again, to find the *principal*, or *base*, when the rate, time, and interest are given.

RULE. — *Divide the given interest by the interest of $1 for the rate and time.*

9. The given interest is $21.87½, rate 7%, time 2½ years; what is the principal? ANS. $125.

Again, find the *principal*, when the amount, rate, and time are given.

RULE. — *Divide the given amount by the amount of one dollar for the rate and time.*

10. If the given amount is $511.75, the rate 6%, and the time 2½yrs., what is the principal?

11. A merchant bought a sloop for $6,250, and sold it at a gain of $1,500; what did he gain per cent.?

OPERATION: The gain is $\frac{1500}{6250}$ of cost. $\frac{1500}{6250} \times 100 =$ 24%.

12. A man spends $1,230 annually, which is 82% of his salary; what is his salary? ANS. $1,500.

13. A man has a capital of $12,500. He puts 15% of it in stocks, 33⅓% in land, and 25% in mortgages; how much of his capital remains?

14. Find the interest on $396.50 for 5 years at 8% per annum.

15. Find the amount of $2,714 for 6 years at 5% per annum.

16. Find the interest of $748 for 8 months at 6%.

17. Find the interest on $1,600 for 4yrs., 5mos., and 12da., at 7% per annum.

18. Find the amount of \$3,000 for 4yrs., 8mos., 6da., at 6% per annum.

19. If \$450 yield \$79.87½ for 3yrs., 6mos., 18da., what is the rate per cent. per annum?

ANS. \$79.87½÷\$15.97½=5%.

Sometimes money is loaned at what is called *annual* interest, which requires simple interest on the principal for a number of years, at a given rate, and also simple interest on each year's interest until paid.

Rule for finding the annual interest of any sum for more than two years: —

1. *Find the simple interest at the given rate and time.*

2. *Find the interest of the first year's interest for one year, and multiply it by the sum of the series, 1, 2, 3, etc., up to the given number of years, less one.*

3. *Add the two results, and the sum will be the annual interest required.*

20. What is the annual interest on \$600 for 3 years at 6%?

OPERATION:

1.	Interest on \$600 for 3 years at 6%,	\$108.00
2.	Interest on \$36 (first year's interest), 2yrs.,	4.32
3.	Interest on \$36 (second year's interest), 1yr.,	2.16
	Total interest,	\$114.48

21. What is the interest on a note for \$980 at 8%, payable in 4 years, with annual interest?

Compound Interest.

Compound interest is the interest on the principal and also on each year's accrued interest, for successive periods. At the end of each period (year or half-year) a *new* principal is formed of the principal and its accrued interest.

22. Find the compound interest on \$600 for 3 years at 6%.

OPERATION: \$600 × 1.06 = \$636 = 1yr. \$636 × 1.06 = \$674.16 = 2yr. \$674.16 × 1.06 = \$714.61 = the amount. The amount \$714.61 − \$600 = \$114.61 = the ANS.

Test Examples in Percentage.

Most of these examples may be solved mentally.

1. What is 1% of 1?

2. What is 1% of \$1?

3. What is 5% of \$20?

4. What is 6% of 200?

5. What is 10% of 600 bushels?

6. What is 50% of 80 pupils?

7. A school of 100 pupils increased 100% in one year; how many pupils then in school?

8. 3 is what per cent. of 6? ANS. $\frac{3}{6} \times 100 = 50\%$.

9. 6 is what per cent. of 3? ANS. $\frac{6}{3} \times 100 = 200\%$.

10. 15 is what per cent. of 20?

11. 18 is what per cent. of 20?

12. 20 is what per cent. of 15? ANS. $\frac{20}{15} \times 100 = 133\frac{1}{3}\%$.

13. My school numbered 90 pupils last year, but it has decreased 10% this year. What is its number now?

14. Suppose the population of Washington to be 180,000, showing an increase of $33\frac{1}{3}\%$. What was the population at the last census?

Section 2. DISCOUNT.

Sometimes a certain percentage is to be deducted from a sum of money, or a bill, note, or draft. This deduction is called *discount*, and the remainder is the *present worth*.

1. To find the *simple discount*, in the common way, multiply the principal by the rate per cent.

2. To find the present worth, subtract the discount from the principal.

1. What is the discount on a bill of \$96, discounted at 6%? Ans. $\$96 \times .06 = \$5.76$.

2. What is the amount due on a bill of \$96, discounted at 6%?

3. If coal is sold at \$6 per ton, and at 10% discount for cash, how much must be paid for 5 tons of coal?

The True Present Worth.

The true present worth of a sum of money, due hereafter, is such a sum as, when put at interest for the given time and rate, will amount to the given sum.

RULE. — *Divide the amount, due at a future time, by the amount of \$1 for the given rate and time, and the quotient will be the present worth; the difference between the quotient and the amount will be the true discount.*

4. Find the *present worth* of \$460, due in $2\frac{1}{2}$ years, at 6%. Ans. \$400.

5. What is the *present worth* of a note for \$481.60, due in 2 years and 6 months?

6. Find the *present worth* and the *discount* of \$750, due in 1yr., 8mos., 12da., at 6%.

7. Find the *present worth* and *discount* on a note for \$1,300, due in 2ys., 8mos., at 7%.

8. Find the *present worth* and *discount* on a note for \$4,800, due in 4 years at 5%.

Bank Discount.

In bank discount, if any time is named in a note, *3 days' grace* is always added to the time, and interest must be reckoned for the 3 days, in addition to the time named. Banks usually require interest *in advance*. To find the proceeds: —

RULE. — *Find the interest, as before directed, on the face of the note for the rate and the time + 3 days' grace, and take this from the face of the note, and the remainder will be the proceeds.*

The following forms are common, (1) for a bank note, and (2) for a negotiable note; and they should be studied and copied for practice.

9.

1. Bank Note.

\$600. *Washington, July* 7, 1885.

Sixty days from date, I promise to pay to the order of Henry R. Miles, Six Hundred dollars, at the National Metropolitan Bank. Value received. Due September 7–10, *at six per cent.*

A. S. RICHARDS.

What is the interest or discount, and how much cash is received on this note?

10. What is the bank discount and present worth of a note for $350, payable in 90 days after date, at 7% ?

11. If $500 are needed in cash to-day, what will be the face of the note, payable at bank in 60 days, at 6% per annum?

OPERATION: The amount of $1 for 63 days is $1.0105. $1.0105 × 500 = $505.25 = ANS.

But if the exact discount is reckoned, the note would be $500 ÷ .9895 = $505.31.

2. Common Negotiable Note.

12.

$500.

Washington, D. C., September 1, 1885.

Ninety days after date, for value received, I promise to pay to the order of C. B. Smith Five Hundred dollars, with interest at six per cent.

B. P. DAVIS.

If the note is to be paid at any bank, or at any fixed place, the place must be named in the note.

Section 3. STOCKS AND BONDS.

A corporation may own Capital Stock, and it may issue Certificates of Stock, which sometimes have both a *par value* and a *market value.* If the *market value* is greater than the *par value*, the stock is said to be *above par*, or at a premium; but if the market value is less than the par value the stock is said to be *below par.*

The terms and language here used, as well as everywhere, should be made perfectly familiar. All the examples under this head are solved upon the principles of percentage, already explained. People who deal in stocks and bonds are generally allowed $\frac{1}{4}\%$ for their service; or some certain per cent. must be fixed, which is called *brokerage*, usually on par value.

1. What is the cost, including brokerage, of 44 shares of bank stock @ $129\frac{1}{2}\%$, if each share is worth $100?

SOLUTION: $\$100 \times 44 = \$4,400$, amount of stock. The brokerage on $\$4,400 \times \frac{1}{4}\% = \$11.00$. Add $\frac{1}{4}\%$ to $129\frac{1}{2}\%$, to include brokerage; then, $\$4,400 \times 1.29\frac{3}{4}$ (or $1.2975) =$ cost $= \$5,709$.

2. Sold $5,000 in gold @ $112\frac{1}{4}\%$. What are the net proceeds, after deducting brokerage?

OPERATION: $112\frac{1}{4} - \frac{1}{4}\% = 1.12$. $1.12 \times \$5,000 = \$5,600 =$ ANS.

3. How much gold @ $111\frac{5}{8}\%$ can be bought for $8,930 in currency, with no brokerage? $\$8,930 \div 1.111\frac{5}{8} = \$8,000$.

4. If 170 railroad bonds cost (including brokerage) $192,100, what is their market value?

EXPLANATION: If a bond at par is $1,000, then $170 \times \$1,000 =$ the par value of the bonds $= \$170,000$. Brokerage at $\frac{1}{4}\%$ amounts to $425. $\$192,100 - \$425 \div \$170,000 = 112\frac{3}{4}\%$.

5. A broker sells 30 shares of bank stock @ $96\frac{1}{2}\%$, and 120 shares of railroad stock at 105%, retaining the brokerage; how much does he pay to the principal?

ANS. $15,456.27.

N. B. — Shares are reckoned here at $100 and bonds at $1,000.

6. The market rate of 5% stock is $85\frac{1}{2}$%; if the purchaser pays brokerage, what rate of interest does he receive on his investment?

OPERATION: $\$5 \div (85\frac{1}{2}\% + \frac{1}{4}\%)$ (brok.) $= \$5 \div .8575 = 5\frac{285}{343}$.

7. I wish to purchase a 5% stock on such terms that it will give me 7% on my investment; how much can I give for the stock, including brokerage?

OPERATION: $\$5 \div .07 = 71\frac{3}{7}\%$.

Note.—To make the operation under the 6th Example clearly understood, we must consider the gain on one share of $100 of stock to be $5. Then, as the purchaser pays the brokerage of $\frac{1}{4}$%, he actually pays $85\frac{1}{2}\% + \frac{1}{4}\% = 85\frac{3}{4}\% =$ $0.8575. Then the whole gain, $5, divided by the value of $1, which is $0.8575, will give the rate per cent.

Again, in the *operation* under the 7th Example, the gain of one share of $100 is $5, and as the stock at 7% includes brokerage, the share is worth only $\frac{5}{7}$ of $\$100 = \$71\frac{3}{7}$, or $71\frac{3}{7}\%$.

Section 4. EXCHANGE.

We must often make payments in distant places by means of drafts, or bills of exchange, which are orders from one party to another to pay a *third* party a certain sum in a specified time.

There are *sight drafts* and *time drafts*. Sight drafts are payable on presentation. When a time draft is used, 3 days of grace are added to the specified time.

The maker of the draft is called the *drawer*. The person who is to pay the draft is called the *drawee*, and the person to be paid is the *payee*.

If the drawee writes *accepted* on the draft over his own name, he is then obligated to pay it.

Par exchange requires a certain sum of money of one country to be equal in value to a specific sum in another country. Thus £1 sterling is equal to $4.8665, in New York. One dollar in Federal money is equal to 5.18 francs, in France, or on exchange.

Form of a Draft.

Washington, August 11, 1885.

At sight, pay to the order of Wm. Windom two hundred and fifty dollars, value received, and charge the same to the account of the National Metropolitan Bank, Washington, D. C.

JOHN SMITH.

1. What is the cost of a sight draft on New York for $1,548; exchange being 2% discount?

OPERATION: $1,548÷1.02=$1,517.64+.

2. What is the cost of a draft on New York for $2,400 in 90 days after sight; interest at 10% and exchange 103%?

OPERATION: Cost of exchange $2,400×1.03=$2,472 for one year. Again, the interest of $2,400 at 10% for 93 days is $62. The difference is $2,472—$62=$2,410=ANS.

Note.—The teacher must explain terms and make the language clearly understood; then give other examples.

3. If exchange is 10½%, how large a sight draft can be bought for $7,900? ANS. $7,900÷1.10½.

4. What is the face of a draft at 30 days, which costs $2,000; exchange being 102%; interest 6%?

OPERATION: The difference between the exchange, 102%, and the interest for 33 days is $1.02—.0055=$1.0145. Then $2,000÷1.045=$1,971.41½=ANS.

Section 5. EQUATION OF PAYMENTS.

Equation of payments is a method of finding the medium or proper time for the payment of different sums, due at different times.

It is based upon the principle that $1, paid with interest one month after it is due, is equivalent to $1 paid with interest one month before it is due.

Thus the use of $5 for 5 months is equivalent to the use of $25 for 1 month. For finding the equated time, we have therefore the following

RULE. — *Multiply each sum by the number of months, or periods of time given (thereby finding a sum for one of the periods of time worth as much as the given sum for all the periods of time), and add the several products. Divide the sum of the products by the amount of the given sums, and the quotient will be the equated time.*

1. A owes $800 payable in 6 months, $600 payable in 8 months, and $1,000 payable in 12 months. What is the equated time for making all the payments at once?

OPERATION:

$800 × 6mo. = $4800 for 1 mo.
600 × 8mo. = 4800 ,, ,,
1000 × 12mo. = 12000 ,, ,,
$2400)$21600(9 months = the equated time.
216
00

But if the credits on the several payments begin at different dates, count the units of time from the date of the first payment. Find the number of days or units of time from this date to the date of each of the other payments, and this will give the units of time for each sum to be paid. Then proceed according to the preceding rule.

2. A owes B $300, payable $80 in 22 days, $100 in 60 days, and $120 in 75 days. What is the equated time for payment?

3. What is the equated time for paying the following sums: $200 now due, $400 in 6 months, and $400 in 15 months?

4. A sells goods to B on a credit of 60 days; namely: January 14, 1885, a bill of $2,000, due March 15; February 10, 1885, a bill of $1,500, due April 11; March 25, 1885, a bill of $3,000, due May 24. What is the equated date of payment?

OPERATION:

$2000 × 0 days =	00000	ANS.
1500 × 27 ,, =	40500	Equated time, 39 days.
3000 × 70 ,, =	210000	,, date, April 23, 1885.
$6500	)$250500(38.538.	

The initial date is March 15; time nothing.
$1,500 runs from March 15 to April 11, or 27 days.
$3,000 runs from March 15 to May 24, or 70 days.
The quotient is *more* than 38½ and is called 39.

5. Mr. Johnson owes Mr. Williams three notes, payable as follows: the 1st for $500, February 12, 1885; the 2d, $400, March 12, 1885, and the 3d for $300, April 1, 1885. When can they all be paid together?

Assume February 12 to be the initial date, then we find $500 × 0 days, $400 × 28 days = $11,200, and $300 × 47 days = $14,100. Sum of products $25,300 ÷ $1,200, sum of payments = 21 days. ANS. March 5.

Note. — The averaging of accounts should be taught in connection with book-keeping; also the methods of accounts in custom-house business, as none but special book-keepers will have such work to do.

Section 6. RATIO AND PROPORTION.

Almost every arithmetical operation required in business transactions involves the principles of ration and proportion.

Thus, what will 6lbs of sugar cost at 11cts. a pound? The proportion stated is: If one pound cost 11cts, what will 6 pounds cost? or as 1 pound is to 6 pounds, so is 11cts. (the cost of 1lb.) to the cost of 6lbs., or 66cts. We wish to buy six times 1lb., so we must pay six times the price of one pound. All operations in proportion are based upon the simple principle thus expressed.

RATIO is the relation of one number to another in respect to the number of times one number will measure another. Usually if two numbers are compared, we take the *first* for the *dividend*, or numerator of a fraction, and the *second* for the *divisor*, or denominator.

A PROPORTION requires an equality of ratios. Several ratios combined may be compared with a single ratio, which is sometimes called double proportion, or double rule of three.

The first term of a ratio is called the *antecedent*, and the second term the *consequent*.

As a complete proportion has two or more complete ratios, it must have *four terms*, so related that the product of the first and fourth terms, or *extremes*, must be equal to the product of the second and third terms or *means*. If, then, we have the product of the means, and *one extreme*, the other extreme can be found by dividing the product of the means by the given extreme.

Again, if we have the product of the extremes, and *one mean*, the other mean can be found by dividing the product of the extremes by the given mean.

A ratio is expressed by placing two dots, like a colon, between the two numbers; thus, 2 : 4 means *as 2 is to 4;*

but when this ratio is compared with another ratio, four dots, or two colons, are placed between the ratios, which means *so is;* thus, 2 : 4 : : 3 : 6, as 2 is to 4 so is 3 to 6.

But if one of these terms is wanting, as 6, the fourth term, it can be found by dividing the product of 3×4, the means, by 2, the first extreme; thus, $\frac{4 \times 3}{2} = 6$.

To state a question we must have the first ratio and the first term of the second ratio.

The corresponding terms of equal ratios must be of the same name or denomination, before exact results can be obtained.

From what has been said above, the following rule is formulated.

RULE. — 1st. *Terms to be compared must be of the same name.*

2d. *The third term of the proportion is to be of the same name as the term sought.*

3d. *Consider from the nature of the question whether the term sought, or fourth term, should be greater or less than the third term; if greater, then place the greater of the two remaining terms for the second term; but if less, then place the less for the second term and the other term for the first term.*

4th. *The fourth term (or answer) will be the quotient of the product of the second and third terms, divided by the first.*

Note. — If the question should contain other ratios, to be compared with the *second* ratio of the proportion, they must be arranged according to the third paragraph in the above rule, and then the continued product of the third term into all the second terms, divided by the continued product of all the first terms, will be the answer.

Exercises for Practice.

1. If the antecedent is 12, and the consequent 6, what is the ratio? $\frac{12}{6}=2=$ the ratio.

2. If the antecedent is 12, and the ratio 2, what is the consequent? $\frac{12}{2}=6$.

3. If the consequent is 6, and the ratio 2, what is the antecedent? $6\times 2=12$.

4. If the antecedent is $\frac{2}{3}$, and the consequent $\frac{1}{6}$, what is the ratio? Get the fractions to the same name and then find the ratio of the numerators. Thus, $\frac{2}{3}=\frac{4}{6}$, then $\frac{4}{6}:\frac{1}{6}$ as $4:1$ $=\frac{4}{1}=4$, the ratio.

5. What is the ratio of £2 to 5s.? £2=40s., then 40s. : 5s. $=\frac{40}{5}=8=$ the ratio.

6. What is the ratio of $22\frac{1}{2}$ to $4\frac{1}{2}$? Ans. $\frac{45}{2}:\frac{9}{2}=$ $45:9=5$.

7. If a man walk 84 miles in three days, how far can he walk in 9 days?

1st. *By Analysis.* He can walk three times as far in 9 days as in three days; hence he will travel $84\times 3=252$ miles.

2d. *By the Rule.* First, place 84 miles for the *third term,* because we wish to find miles. Then, as the demand is on 9 days, and the condition is on three days, we see that he can travel further in 9 days than in 3 days, therefore, the greater term, 9, must be in the second term, and 3 in the first; thus, $3:9::84:$ Ans. $=\frac{9\times 84}{3}=\frac{756}{3}=252=$ Ans.

8. If 12 men can build a wall in 20 days (condition), how many men can build it in 5 days?

Ans. $5:20::12:48$.

9. If a piece of cloth 20 yards long and $\frac{3}{4}$ yard wide is required to make a dress (condition), how wide must a piece of cloth be, which is 12 yards long, to make the same dress?

Here width is to be found; place $\frac{3}{4}$, then, for third term. Now will 12 yards in length require more or less width to make a dress than 20 yards? Plainly, more; then put 20, the greater, in the second place, and 12 in the first; thus, $20 : 12 :: \frac{3}{4}$: ANS.

10. If a man can do a piece of work in 20 days, working 10 hours a day, how long will it take him to do the same work working 12 hours a day? ANS. $16\frac{2}{3}$ days.

11. If it cost $40 to board 3 men 5 weeks, what will it cost to board 12 men 10 weeks?

Here two ratios are to be combined to find the *second* ratio. We must find cost. $40 must be the 3d term. 12 men cost more than 3 men, and 10 weeks will cost more than 5 weeks.

$$
\begin{array}{ll}
\text{STATEMENT}: & 3 : 12 :: \$40 : \text{ANS.} \\
 & 5 : 10
\end{array}
$$

12. If 20 men, working 11 hours a day for 30 days, can earn $3,300, how much can 36 men earn in 40 days, working 10 hours a day?

$$
\begin{array}{l}
{}^{1}\,\cancel{20} : 36 :: \cancel{3300} : \quad \text{ANS. } 7,200. \\
{}^{1}\,\cancel{30} : \cancel{40}\ {}^{2} \quad \cancel{300}\ {}^{10} \\
{}^{1}\,\cancel{11} : 10 \\
\quad 10 \times 10 \times 36 \times 2 = 7,200.
\end{array}
$$

In finding the fourth term the process of solution may be shortened by cancellation. Observe that all the terms of the means are factors of a product which is to be divided by the product of the terms in the first extreme; therefore, all the *common factors* which make each product can be canceled, and the uncanceled factors used according to the rule. (See the operation above.)

13. If a man can walk 250 miles in 9 days of 12 hours each, how many days of 10 hours each will it take him to walk 400 miles?

14. Divide \$540 between a man, his wife, and three children, so that the wife shall have twice as much as each child, and the man twice as much as the wife.

ANS. \$60, \$120, \$240.

15. A banker owes A \$500, B \$750, C \$900, and D \$1,250, but his estate is worth only \$1,020. What can each man receive?

16. A and B start from places 150 miles apart and travel toward each other. A travels 7 miles an hour, and B 8 miles an hour. How far does each travel before they meet?

17. If 24yds., 3qrs. of carpeting, 1 yard wide, will cover a room, how many yards $1\frac{3}{4}$ yards wide will cover the same room? How many if $\frac{3}{4}$ yard wide?

18. If 12 boys pay \$2,000 for one year's tuition, what must 14 boys pay for $18\frac{1}{2}$ months' tuition?

19. At what time between one and two o'clock are both hands of a clock together? Statement, 55 : 60 : : 60 : ANS.

20. A farmer puts his whole flock of sheep into 3 pastures; $\frac{1}{3}$ into one pasture, $\frac{1}{2}$ into another, and 32 into a third pasture. How many in each pasture?

21. If 14 meters of cloth can be bought for 350 francs, how many meters can be bought for 875 francs?

Section 7. MEDIAL PROPORTION, ETC.

It is necessary sometimes to find an average, or mean proportion, between two or more numbers.

1. To find a mean, or average, number between two numbers, take one half their sum.

2. To find an average among any number of numbers, divide the sum of all the numbers by the number of numbers.

3. To find the price of a mixture of simples when the price and quantity of each simple are given.

1st RULE. — *Multiply the number of parts of each simple by its price, and divide the sum of the products by the sum of all the parts. Thus, 5lbs. at 10cts., 6lbs. at 12cts., 7lbs. at 14cts., will make 18lbs. at $2.20, or 12⅑cts. a pound.*

4. To find the quantity of each simple when its price and the price of the required mixture are given.

2d RULE. — 1. *Write the prices of the simples, one under the other, beginning with the least, and place the mixture price on the left.*

2. *Compare each simple price with the mixture price.*

3. *If the difference between the* mean price *and the price of a simple exceeding the mean price is greater than the difference between the mean price and the price of a simple which is below the mean price, then the larger difference will be the proportion of that simple nearest in value to the mean price, and should be placed opposite to it; and the* less difference *should be placed opposite to the other simple compared.*

But if the price of two or more *simples is greater or less than the mean price, the sum of the differences will be the proportion of the simple with which they are connected. These differences will be the required quantity of each simple opposite to them.*

1. If one kind of wine at 16s. is mixed with another kind at 18s., and with another kind of wine at 22s., so that the mixture is worth 20s., what proportion of each must be taken?

20	16s.	2=2
	18s.	2=2
	22s.	4+2=6

OPERATION: Here, two simples are less than the mean price, and as the sum of these differences between them and the mean price is more than the difference between the other simple and the mean price, the sum of them must be placed opposite to that simple which is nearest, but above, the mean price.

2. Mix brandy at 12s., wine at 10s., cider at 1s., and water at 0s. per gallon, so as to make a mixture worth 8s. per gallon.

8s.	0s.	=4 water,
	1s.	=2 cider,
	10s.	=7 wine,
	12s.	=8 brandy.

5. To find how much of each simple must be mixed to make a mixture of a given price when the prices of several simples and the *quantity* of one of them are given.

3d RULE. — *Find the proportions by Rule 2, as in the last example; then say: As the proportional part opposite to the given quantity is to the given quantity, so is any other proportional quantity to its required quantity.*

3. If in the last example 40 gallons of brandy were to be mixed with the wine, cider, and water, how much of each of the other simples must be taken? ANS. 8 : 40 : : 7 : 35.

6. To find the proportional quantity of each simple when the *amount* of the mixture, and the prices of the simples, and the mean price are given.

4th RULE. — *Find the proportional quantities as before, and then say: As the* sum *of the proportional quantities thus found is to the required amount of the mixture, so is each proportional quantity to its required quantity.*

4. If in the last example the amount of the mixture is to be 105 gallons, what will be the proportional parts of this amount? ANS. 21 : 105 : : 8 : 40.

Section 7. POWERS AND ROOTS.

1. Powers.

The *first* power of any number is the number itself; the *second* power of any number is the number multiplied by itself, or taken as a factor *twice;* and the *third* power of any number is the number taken as a factor three times.

Thus, 1 is the *first* power of 1; and one is also the *second* and *third* power of 1. As $1 \times 1 \times 1 = 1$.

2 is the first power of 2, but the second power of 2 is $2 \times 2 = 4$; and the third power, or cube, of 2 is $2 \times 2 \times 2 = 8$. The *power* of any number is known by the number of times it is used as a factor; and the process of raising a number to any power is called *involution.* Usually a figure indicating the power is placed just at the right and above the number; thus, $3^2 = 9$, $3^3 = 27$, $2^3 = 8$.

EXAMPLES FOR MENTAL SOLUTION.

Find the powers of the following numbers as indicated by their *exponents:* 5^2, 3^3, 6^2, 6^3, 3^4, 4^3, 5^3, 6^2, 7^2, 8^2; also, $\frac{1}{2}^2$, $\frac{1}{2}^3$, $\frac{3}{4}^2$, $\frac{3}{4}^3$, $\frac{2}{3}^2$, $\frac{2}{3}^3$. If fractions are to be raised to a power, involve both the numerator and the denominator to the required power.

2. Roots.

The *first* root of any number is the number itself. The second or square root of any number is that number which when squared produces the given number; and the *third* or cube root of any number is that number which, taken as a factor three times, will produce the given number.

EXAMPLES FOR MENTAL SOLUTION.

Find the root of the following numbers indicated by their exponents. There are two methods of expressing roots: *First*, by a hook in front of a figure or number, thus, $\sqrt{4}=2$; or, *second*, $4^{\frac{1}{2}}=2$. $\sqrt[3]{8}=2=8^{\frac{1}{3}}=2$.

$\sqrt{1}$, $\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{36}$, $\sqrt{49}$, $\sqrt{64}$, $\sqrt{81}$; or, $\sqrt[3]{8}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$; or, $\sqrt{\frac{4}{9}}$ = the square root of both numerator and denominator; also, $\sqrt{\frac{9}{16}}$, $\sqrt{\frac{16}{25}}$, $\sqrt{\frac{25}{36}}$, etc.; or, $\sqrt[3]{\frac{8}{27}}$, $\sqrt[3]{\frac{27}{64}}$, $\sqrt[3]{\frac{64}{125}}$, etc.; $\sqrt[3]{216}$, $\sqrt[3]{343}$, $\sqrt[3]{512}$, $\sqrt[3]{729}$. The corresponding roots are the 9 digits $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, etc.; or, $(\frac{9}{16})^{\frac{1}{2}}$, $(\frac{16}{25})^{\frac{1}{2}}$, $(\frac{25}{36})^{\frac{1}{2}}$.

In squaring any one of the 9 digits we find that when 1, 2, and 3 are squared they produce only *one* figure; and that no one of the remaining digits, *squared*, produces more than *two* figures.

The square root of any complete square, less than 100, will always contain one of the digits; but if the number has *three* or *four* figures, its square root must contain *two* figures; and if the number has *five* or *six* figures its square root must contain *three* figures. A period, therefore, contains either one or two figures; two periods must have three or four figures; and three periods must have five or six figures, and so on.

GENERAL RULE FOR FINDING THE SQUARE ROOT.

1st.—Determine the number of periods, which will show the number of figures in the root.

2d.—Find by inspection or trial the largest of the digits whose square will come nearest in value to the left-hand period, and place it in the first place of the quotient.

3d.—Find next, by trial, such figure annexed to the first figure of the root just found as will, when these two figures are squared, come nearest to the value of all the figures in the *first and second periods; and place this figure in the second place of the root.*

4th. — If there is another period, find by trial the third *figure of the root, so that the whole root, squared, will come nearest in value to the figures in the three periods.*

5th. — If there are other periods, proceed in the same manner.

Note. — If there should finally be a remainder, a more exact root can be found by annexing a period of ciphers and getting the root as before, until sufficient accuracy is attained.

1. Find the $\sqrt{81}$.

2. Find the $\sqrt{121}$.

3. Find the $\sqrt{144}$.

4. Find the $\sqrt{169}$.

5. Find the $\sqrt{196}$.

6. Find the $\sqrt{225}$.

As it is not always easy to find the square root of large numbers by simple inspection, the solution of the following question, involving four figures, or two periods, will aid in making the above-described method clear.

7. What is the square root of 1,024?

OPERATION: Here are four figures, and consequently two periods. The first left-hand period is 10, and the greatest square in it is 9; and the square root of 9 is 3, which is therefore the first figure in the root. As there are but two periods, this figure is in *ten's* place. Now find by inspection the number, which, when annexed to the three tens, and with it squared, will equal in value the whole number, 1,024, or will come nearest to it. This result or quotient is the square root. Thus, $\sqrt{1024}=32$.

8. What is the $\sqrt{2304}$? The largest complete square in 23 is 16, and its square root is 4, or 4 tens, or 40. $40^2=$ 1,600. We now find a remainder of 704, so that the root *forty* (40) is not large enough to make a power equal to 2,304, and we must increase 40 until the sum squared shall equal the given number, or come as near to it as possible.

9. What is the $\sqrt{9801}$? The greatest square in 98 is 81, and its square root 9. Now, by trial, annex 9, and square the whole; thus, $99^2=9,801$.

10. What is the $\sqrt{50625}$? Here are three periods — 5 is the left-hand period, or 5,00,00. The greatest square in 5 is 4, and its square root is 2 (or 200). Now find a figure, which, when it is annexed to 2, and with it squared, will come nearest to the first and second periods, 5,06. This is 2, or 2 tens. $22^2=484$, or 4,84,00. Next find a figure to annex to 22 tens, which, with them squared, will come nearest to the whole number, 50,625. This figure is 5, making 225, and $225^2=50,625$.

Note. 1. — If it is found that any trial figure is not large enough, or is too large, increase or diminish it until its square comes as near as possible to the desired number.

Note. 2. — Bear in mind that in a complete square, if the last figure is 1, then the last figure in the root will be 1 or 9; or, if the last figure is 4, then the last figure in the root will be 2 or 8; or if it is 5, the last root figure will be 5; or, if 6, then the root figure will be 6; or, if 9, the last root figure will be 3 or 7. These suggestions are only necessary *to aid* in finding the proper root figures.

For all practical purposes the above directions are sufficient to secure accurate and rapid results.

The common rule, which requires trials, multiplication, division, and subtraction, is as follows: —

RULE. 1st. — *Point off the number into periods of two figures, beginning with units.* (Reason given above.)

2d. — *Find the greatest square in the left-hand period, which may not be more than one figure. Take its square root for the first figure of the root. Subtract the square thus found from the left-hand period, and to the remainder annex the next period for a dividend.*

3d. — *Find a trial divisor by doubling the root found, and then find how many times it is contained in the dividend exclusive of the right-hand figure. Write the quotient for a trial figure of the root, and annex it to the trial divisor.*

4th. — *Multiply the divisor thus completed by the trial figure of the root just found; subtract the product from the dividend and to the remainder annex the succeeding period for a new dividend (provided the remainder found is less than the divisor).*

5th. — *Proceed in the same way until all the periods have been used.*

Note 1. — If there should be still a remainder after all the periods are brought down, annex a period of ciphers, and proceed as with whole numbers.

Note 2. — To find the square root of a common fraction, get the root of the numerator and of the denominator separately.

11. Find the $\sqrt{\frac{16}{25}} = \frac{4}{5}$.

12. Find the $\sqrt{1\frac{44}{169}} = 1\frac{2}{13}$.

13. A general forms an army of 17,649 men into a square. How many men are in each rank, and how many ranks in the square?

14. A square field contains 160 acres. What will it cost to build a wall around it if each rod of wall costs $2?

Cube Root.

The cube root of any number has been defined to be that number which taken as a factor three times will produce the given number. We have also found the cubes of each of the nine digits to be as follows, assuming each digit to be the cube root of some number: —

1. Digit roots, 1, 2, 3, 4, 5, 6, 7, 8, 9.
2. Digit cubes, 1, 8, 27, 64, 125, 216, 343, 512, 729.

These digit cubes should be memorized.

The cube root of any number from 1 to 999 must always be found by inspection or trial.

Geometrically a cube is a solid having six equal and square sides. The length of one of these sides is the cube root of the solid contents. To find the solid contents of a cube, multiply its length, breadth, and thickness together. In cubing the digits, the cubes of 1 and 2 have each two figures, namely, 27 and 64; and the cubes of each of the other digits will have three, and never more than three, figures; consequently, counting from units toward the left, every three figures will have one figure in the cube root, but the last period to the left may have one, two, or three figures.

The following method of extracting the cube root of any number having more than one period is purely arithmetical, and it is believed to be more expeditious, and more readily understood, than the rule in common use.

Rule 1. — *Find the number of cubical periods, which will show the number of figures in the root.*

2. — *Find by trial the greatest cube in the left-hand period and place its root in the quotient's place for the first figure in the root.*

3. — *Next, take the two left-hand periods and find* by trial *the number to be annexed to the first figure of the root just*

found, which, when both are cubed, will come nearest in value to the value of the figures in the first two periods.

4. — *If any other periods are to be used, find the next root figure, which, when annexed to the preceding root figures, and cubed, will come nearest to the value of the three periods taken.*

5. — *If there are more than three periods, proceed in the same manner as before.*

Note 1. — If there be a remainder, annex to the whole number one or more periods of ciphers for decimal parts of the root.

Note 2. — In finding the trial roots assistance may be found by comparing the last figure of the cube of any one of the digits with the last figure of the last period used.

Note 3. — Any digit with a cipher annexed, when cubed will be found to be the cube of the digit with *three ciphers* annexed. Thus, $60^3=216{,}000=6^3$ with three ciphers.

1. What is the cube root of 4,096, or $\sqrt[3]{4096}$?

OPERATION: Here the periods are two, and the greatest cube in 4 is 1, which is the first root figure. Now find by trial the root figure to be annexed to 1. Observe that the last figure in the given number is 6, and that the last figure in the cube of the digit 6 is also 6; hence it may be presumed that the last figure in the root sought is 6, making the whole root 16; which, cubed, is 4,096.

2. What is the $\sqrt[3]{35937}$? Here are two periods, of course. The greatest cube in 35 is 27, and its root 3, the first figure in the root. The last figure in the example is 7, and 3 is the only digit, which, when cubed, has seven for its last figure, and is therefore probably the last figure in the root=33.

3. What is the $\sqrt[3]{403583419}$? Three periods, of course.

N. B. — The common rule for extracting the cube root is here given, to be used if desirable.

RULE 1. — *Find the number of periods as before.*

2. — *Find the greatest cube in the left-hand period and place its root in the quotient's place for the first figure of the root. Subtract the cube from the first period and annex to the remainder the next period for a dividend.*

3. — *Multiply the square of the root thus found by 3, and to the product annex two ciphers for a trial divisor; find how many times this is contained in the dividend and write the quotient for a trial figure of the root; then, annex this trial figure to three times the root previously found; multiply this result by the trial figure and add the resulting product to the trial divisor, for a complete divisor.*

4. — *Multiply the divisor thus completed by the trial figure of the root; subtract the product from the dividend and to the remainder annex the following period (if any) for a new dividend.*

5. — *Proceed as before with any remaining periods.*

Note 1. — If there is not an exact root, annex a period of ciphers to the remainder and proceed as before, for any decimal parts of the root sought.

Note 2. — The cube root of a common fraction is the cube root of the numerator divided by the cube root of the denominator.

The teacher can supply any number of test examples.

Section 8. PROGRESSION.

Arithmetical Progression.

Progression treats of a series of numbers which increase by a regular addition, or decrease by a regular subtraction.

Such a series is called *arithmetical*. The nine digits, in their common order, make a series in arithmetical progression; thus, 1, 2, 3, 4, 5, 6, 7, 8, 9. This is an *ascending series;* but in a reversed order they become a *descending series*, whose *common difference* is 1. The common difference may be any number; thus, in the series 2, 4, 6, 8, 10, the common difference is 2; and in the series 3, 6, 9, 12, it is 3.

1. Find the sum of the nine digits.

Observe that here are nine terms, 1, 2, 3, 4, 5, 6, 7, 8, 9, and that the mean or middle term is 5, which equals half the sum of the extremes; thus, $1+9=10$. $10 \div 2=5$.

The sum of this series can be found by adding the numbers; but the same result may be attained by multiplying this mean term 5 by the number of terms 9; thus, $5 \times 9=45$. Hence the sum of a series of terms in arithmetical progression can be found by multiplying half the sum of the extremes by the number of terms.

2. What is the sum of the following series: 2, 4, 6, 8, 10, 12, 14?

Here are seven terms, and the middle term is 8, and is found by taking half of $2+14=\frac{16}{2}=8$. Observe here that the common difference, 2, is added to 2 six times; so that $2 \times 6+2=14$, the last term. Hence, if we have the common difference, the first term, and the number of terms, we can find the last term of an arithmetical series by multiplying the common difference by the number which is one less than the number of terms, and adding the first term.

3. If the first term be four, the number of terms 8, and the common difference 4, what is the last term, and what is the sum of all the terms?

$4+(4 \times 7)=32=$ the last term, and, $\frac{4+32}{2} \times 8=144=$ sum of the terms.

4. If the first term be 38, the number of terms 8, and the common difference 5, what is the last term, and sum of all the terms?

5. If the first term is 7, the common difference 4, and the number of terms 7, what is the last term, and sum of all the terms?

Geometrical Progression.

But progression also treats of a series of numbers which increase by a regular multiplier and decrease by a regular divisor. This is called GEOMETRICAL PROGRESSION. Thus, 1, 3, 9, 27, 81; or, 81, 27, 9, 3, 1; or, $1\times3\times3\times3\times3=81$; or, $1\times3^4=81$.

The number used as a common multiplier, or divisor, is called the RATIO.

In an ascending series the ratio equals the quotient of the second term divided by the first; but in a descending series the ratio equals the quotient of the first term divided by the second.

TO FIND ANY TERM IN A SERIES when the *first* term, the *ratio*, and *number* of terms are given.

1st. In an ascending series multiply the first term by the ratio for the second term; and then, the second term by the ratio for the third term, and so on, as many times, *less one*, as there are terms in the series; or, repeat the ratio as a factor as many times as the number of terms less one, and multiply the result by the first term.

2d. In a descending series divide the first term by the ratio, and then the quotients continuously, as many times, less one, as the number of terms; or, divide the first term by the ratio raised to a power one less than the number of terms.

1. If the first term be 2, the ratio 3, and the number of terms 6, what is the last term?

2. If the first term be 3, the ratio 4, and the number of terms 5, what is the last term?

3. If the first term of descending series be 135, the ratio 3, and the number of terms 4, what is the last term?

ANS. $135 \div 3^3 = 5$.

4. If the first term be 192, the ratio 2, and the number of terms in a descending series 7, what is the last term?

TO FIND THE SUM OF A GEOMETRICAL SERIES.

1st Method.—Find each term in the series and then add all the terms.

2d Method.—Multiply the last or largest term by the ratio; from which subtract the first or least term, and divide the remainder by the ratio less one, and the quotient will be the sum of all the terms.

5. ILLUSTRATION: Take the series 2, 6, 18, 54, 162. The first term is 2, the number of terms 5, and the ratio 3. If the whole series be multiplied by the ratio 3, the result will be as follows: $(2, 6, 18, 54, 162) \times 3 = 6, 18, 54, 162, 486$. The sum of the last series is plainly three times as much as the sum of the first series. Subtract the first series from the second and the result will be the difference of the extremes: namely, $486 - 2 = 484$; which must, of course, be *twice* the sum of the first series. Divide this difference, in this case, by 2, which is the same as the ratio (3) less 1, and the quotient must be once the sum of the first series, or sum of all the terms. ANS. 242.

6. If the first term is 3, the ratio 2, and the number of terms 5, what is the sum of all the terms?

First find the last term and then the sum of the terms.

7. What is the sum of all the terms when the number of terms is 7, the first term 2, and the ratio 3?

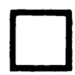
8. How large a debt may be paid in 12 months, by paying $1 the first month, $4 the second month, $16 the third month, and so on, in a geometrical ratio, at the end of 12 months? Ans. $5,592,405.

Section 9. MENSURATION.

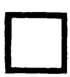
All kinds of surfaces and solids which are formed by straight lines and regular curves can be measured; and the operation of finding the contents of such surfaces and solids is called Mensuration.

A few of the more common and practical cases will here be given.

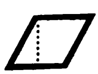
Case 1. — The square has four equal sides and four right angles. Its contents are found by squaring the length of one side.



Case 2. — The parallelogram is a four-sided figure having its opposite sides equal and parallel and its opposite angles equal. A right-angled parallelogram is called a *rectangle*, and an oblique-angled parallelogram is called a *rhomboid*. A *rhombus* has *four* equal sides, with opposite sides parallel, and its opposite angles equal.



Square.



Rhombus.

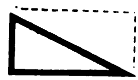
Rectangle.

Rhomboid.

The contents of a rectangle are found by multiplying its length by its breadth.

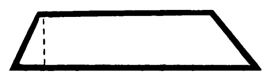
The contents of a rhombus and rhomboid are found by multiplying their length by their altitude.

The altitude of any rectilinear figure is the shortest distance from one of its angles to the opposite side.



Case 3. — The triangle is a figure having three sides and three angles. A right-angled triangle has one right angle, and its contents are obtained by multiplying its base by half its perpendicular, or half the base by the perpendicular.

An oblique-angled triangle may have one obtuse angle, or three acute angles; and its contents are obtained by multiplying its base by half its altitude.



Case 4. — The trapezoid has four sides, only two of which are parallel. The contents are found by multiplying half the sum of the two parallel sides by its altitude.

1. (Under Case 1.) What are the contents of a parcel of land 40 rods square?

2. (Under Case 2.) *First*, find the contents of a rectangle whose length is 40 rods and width 25 rods; *second*, of a rhombus whose sides are 20 rods and whose altitude is 15 rods; and *third*, of a rhomboid whose length is 40 rods and its altitude 30 rods.

3. (Under Case 3.) *First*, find the contents of a right-angled triangle whose base is 20 rods and its height 28 rods; *second*, of an oblique-angled triangle whose base is 40 rods and its altitude 25 rods.

4. Find the contents of a trapezoid whose parallel sides are respectively 20 rods and 36 rods, and its altitude 15 rods.

The Circle.

CASE 5. — A circle is a figure bounded by a curved line, every part of whose circumference is equally distant from a point within called a centre.

Its diameter is a straight line passing through its centre and meeting its circumference. The radius is *half* of the diameter.

1. The contents or area of a circle may be found if the diameter is known, first, by multiplying the square of its diameter by the decimal .7854; or, *second*, by multiplying the square of its radius by 3.1416.

2. If the circumference of a circle is known, the diameter is found by dividing the circumference by 3.1416.

3. If the diameter is known, the circumference is found by multiplying the diameter by 3.1416.

1. What is the area of a circle whose diameter is 5 rods and its radius $2\frac{1}{2}$ rods?

OPERATION: First, $5^2 \times .7854$; or, second, $2.5^2 \times 3.1416$.

2. The diameter of a circular garden is 150 feet. What are its circumference and area?

3. The diameter of a circle is 40 rods. What are its circumference and area?

4. The circumference of a circle is 22 feet. What are its diameter and area?

5. The area of a circle is 176.715 square feet. What are its diameter and its circumference?

OPERATION: $\sqrt{176.715 \div 3.1416} = 7.5 =$ its semi-diameter, or radius.

The Sphere.

CASE 6. — The sphere is a solid ball every point of whose surface is equally distant from a point within called the centre.

1. The surface of a sphere is equal to 4 times the square of its radius, or semi-diameter, multiplied by 3.1416.

2. The diameter of the sphere is equal to the length of a straight line passing through its centre from one surface to another.

3. The solid contents of a sphere, or ball, may be found by multiplying its surface by one third of its radius, or by cubing the diameter and multiplying this cube by the decimal .5236.

1. The diameter of a sphere is 8 feet. What is its surface? Radius $4^2 \times 4 \times 3.1416 =$ Ans.

2. How many square feet of tin will cover the surface of a globe 12 inches in diameter?

Operation: $6^2 \times 4 \times 3.1416 \div 144 =$ Ans.

3. (Under Note 3.) What is the solidity of a sphere whose diameter is 20 feet?

Parallelopipedons, Prisms, Cylinders, Etc.

Case 7. — All these solids should be clearly understood.

The contents of all rectangular solids can be found by multiplying their length, breadth, and thickness together.

The contents of all prisms can be found by multiplying the area of their bases by their height or altitude.

Prisms are triangular, rectangular, hexagonal, rhomboidal, etc.

The surface of a cylinder can be found by multiplying its circumference by its height and adding the product to the sum of the areas of its two circular ends.

1. The side of a cube is 2 feet; 1st, what is its surface? 2d, what are its solid contents?

2. Find, *first*, the surface of a right prism having four equal sides, each 3 feet wide and 8 feet long; and *second*, the solid contents.

3. Find the entire surface of a prism of four equal sides, whose width is 3 feet each and the length is 5 feet; then find the solid contents.

To find the area of a regular polygon whose perimeter is known, and the perpendicular distance from its centre to one of its sides.

RULE. — *Multiply the perimeter of the figure by half the perpendicular distance from the centre to one side.*

4. What is the area of a regular hexagon whose sides are 14.6, and the perpendicular from the centre to a side 12.64 feet? $(14.6 \times 6) \times \frac{12.64}{2} =$ ANS.

5. What is the entire surface of a right cylinder whose length is 12 inches and the diameter of the base is 8 inches?

First find the circumference $= 8 \times 3.1416$; then the convex surface equals $8 \times 3.1416 \times 12$ inches. Now find the sum of the areas of the two ends, which equals $8^2 \times .7854 \times 2$, and add it to the convex surface. ANS. 402.1048 sq. inches.

6. How deep must a circular cistern be whose diameter is uniformly 8 feet, in order that it may contain 50 hogsheads of water, if a gallon contain 231 cubic inches?

Pyramids and Cones.

CASE 8. — The solid contents of a pyramid are one third ($\frac{1}{3}$) of those of a prism of the same base and altitude; and the solid contents of a cone are $\frac{1}{3}$ of those of a cylinder of the same base and altitude.

RULE to find the solid contents of a pyramid or of a cone: *Multiply the area of the base by one third of the altitude.*

1. The altitude of a cone is 18 feet and the radius of its base 4 feet. What are its contents?

2. Find the entire surface of a cone whose side or slant height is 36 feet and its diameter 18 feet.

The entire surface is equal to the product of the perimeter or circumference of the base by half the slant height plus the area of the base. In Example 2 first find the circumference of the base, which equals 18×3.1416 multiplied by $\frac{1}{2} \times 36 = 18$. To the product add the area of the base.

3. The circumference of the base of a cone is 10.75 feet, and the slant height is 18.25 feet. What is its entire surface?

A pyramid may have a triangular, quadrangular, pentagonal, or hexagonal base, with triangular sides meeting at a common angle.

The slant height of a pyramid, or of a cone, is the shortest distance from its common angle to the perimeter of its base.

The altitude of a pyramid or cone is the shortest distance from its common angle or apex to the plane of its base.

1. What is the entire surface of a square pyramid each side of whose base is 3 feet, and the slant height 24.05 feet?

ANS. 153.3 square feet.

2. If one of the largest of the Egyptian pyramids is 477 feet in slant height, and each side of its base, which is square, is 720 feet, what are the contents in solid yards?

ANS. 2,003,200 cu. yd.

3. How many solid feet in a regular hexagonal pyramid whose altitude is 36 feet, and whose sides are $14\frac{1}{2}$ feet, and the perpendicular from the centre of the base to a side is 12.64 feet?

OPERATION: $14\frac{1}{2} \times 6 \times \frac{12.64}{2} \times \frac{36}{3} =$ ANS. 6,598.08 feet.

Measurement of Boards, Timber, Etc.

CASE 9. — A *board-foot* is a board one foot square and one inch or $\frac{1}{12}$ of a cubic foot thick.

Sometimes a board is wider at one end than at the other. Find the average width by taking half of the sum of the widths of the two ends.

The length is usually in feet and the width in inches. To find the contents in board-feet: *Multiply the length in feet by the width in inches and divide the product by 12.*

1. A plank is 16 feet long and 16 inches wide. How many board-feet? $\frac{16 \times 16}{12} = 21\frac{1}{3} =$ ANS.

2. A plank is 16 feet long, and 16 inches wide at one end and 18 inches at the other; how many board-feet?

3. A plank is 15 feet long, 12 inches wide at one end and 14 inches at the other, and $2\frac{1}{2}$ inches thick. How many board-feet?

Round and Hewn Timber.

When timber is not measured by board measure, round timber may be measured by finding the girt or circumference of the *mean cross-section*, taking off the bark. The cross-section in square inches may be found by the following

RULE. — *Multiply the square of the girt in inches by .0796.*

In square, or hewn, timber, find the square inches in the *mean cross-section* by multiplying the breadth by the thickness.

Knowing the area of the two end sections and of the mean section, in square inches, and the length in feet, we can find the number of cubic feet in a stick of timber by the following

RULE. — *To the sum of the two end sections add four times the mean section, all in square inches, and multiply the result by the length in feet; then divide by 864, and the quotient will be the number of cubic feet required.* Or,

To find the solid contents of a round, tapering stick of timber: Find the *average girt* and divide it by 5. Multiply the square of one fifth of the girt by *twice the length* and the product will be the solidity nearly.

4. Required the contents of a piece of timber 9 feet, 6 inches long, its average girt being 14 feet.

OPERATION: $\frac{14}{5}^2 \times 9\frac{1}{2} \times 2 = 2.8^2 \times 19 = 148.96.$

5. If a tree girts 14 feet at one end and 2 feet at the other, and is 24 feet long, how many solid feet does it contain?

OPERATION: $\frac{14+2}{2} = 8. \quad \left(\frac{8}{5}\right)^2 \times 48 = 122.88.$

Method of Using Duodecimals.

This method might have been introduced under denominate numbers. The denominations are feet, primes (or inches), seconds, thirds, etc. The foot is divided into *twelve* equal parts called *primes;* each prime is divided into *twelve* equal parts called *seconds*, and each second into *twelve* equal parts called *thirds*, etc.

Table.

12	Fifths ($''''''$)	make	1	Fourth.	($''''$)
12	Fourths	,,	1	Third.	($'''$)
12	Thirds	,,	1	Second.	($''$)
12	Seconds	,,	1	Prime.	($'$)
12	Primes	,,	1	Foot.	

These numbers can be multiplied without reducing them all to one name. Feet multiplied by feet produce feet; feet multiplied by primes produce primes; primes multiplied by primes produce seconds; primes multiplied by seconds produce thirds; and any term of one denomination multiplied by any other term produces a product whose name will be indicated by the sum of the signs of the factors; thus, $3'' \times 4''' = 12'''''$.

1. What is the product of 2ft., $6' \times 6$? 12ft., $36' =$ 15ft.

2. Multiply 3ft., $6'$ by 4ft., $3'$. Thus, $18'' = 1'$, $6''$, and $33' =$ 2ft., $9'$.

3ft.	$6'\times$	
4ft.	$3'$	
	$9'$	$18''$
12ft.	$24'$	
14ft.	$10'$	$6''$

3. Multiply 3ft., $4'$, $2''$ by 4ft., $6'$.

OPERATION:

3ft.	$4'$	$2''\times$	
4ft.	$6'$		
	$18'$	$24''$	$12'''$
12ft.	$16'$	$8''$	
15ft.	$0'$	$9''$	$0'''$

4. Find the continued product of 3ft., $4'$, 2ft., $7'$, and 6ft., $11'$.

5. Find the continued product of 4ft., $3'$, 5ft., $2'$, and $3'$, $4''$.

Miscellaneous Test Questions.

1. If $\frac{5}{8}$ of a ton of guano cost $49, what will 1cwt. cost? ANS. $3.92.

2. How many bricks, 8in. long, 4in. wide, and 2in. thick, will be required to build the walls of a house 80ft. long, 40ft. wide, and 25ft. high, besides the thickness of the mortar, — the walls to be 12 inches thick? ANS. 159,300.

3. A merchant sold a piece of cloth for $24 and thereby lost 25%. What per cent. would have been the gain had he sold it for $34? ANS. $6\frac{1}{4}$%.

4. Bought a hogshead of molasses containing 120 gallons, for $30; but twenty gallons leaked out. What must the rest be sold for by the gallon to gain $10? ANS. 40cts.

5. If a sportsman spend $\frac{1}{3}$ of his time in smoking, $\frac{1}{4}$ in gunning, 2 hours daily in loafing, and six hours in eating, drinking, and sleeping, how much remains for useful purposes?

6. If a lady spend $\frac{1}{4}$ of her time in sleep, $\frac{1}{5}$ in making calls, $\frac{1}{6}$ at her toilet, $\frac{1}{7}$ in reading novels, and two hours each day in receiving calls, how large a portion of time remains for improving her mind and taking care of her house? ANS. $3\frac{27}{35}$ hours.

7. If a staff 4 feet long cast a shadow $5\frac{3}{5}$ feet, what is the height of a steeple whose shadow is 150 feet at the same time? ANS. $107\frac{1}{7}$ feet.

8. If $\frac{3}{11}$ of a yard cost $5, what quantity will $17.50 purchase? ANS. $\frac{21}{22}$ yard.

9. A certain town is taxed $6,045.50; the valuation of the town is $293,275. There are 150 polls in the town,

taxed at \$1.20 each. What is the tax on \$1, and what does a man pay who has four polls and whose valuation is \$3,675? Ans. 2cts., and \$78.30.

10. How many yards of paper 30 inches wide will it require to cover the walls of a room $15\frac{1}{2}$ feet long, $11\frac{1}{4}$ feet wide, and $7\frac{3}{4}$ feet high? How many pieces or rolls of paper if they are 6 yards long? Ans. $55\frac{17}{60}$yds., and $9\frac{77}{360}$ of rolls.

11. A bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep. What should be the dimensions of a similar measure that contains four quarts?

12. A gentleman willed $\frac{1}{3}$ of his estate to his wife, $\frac{1}{4}$ of the remainder to his oldest son, and $\frac{1}{6}$ of the residue, which was \$151.33$\frac{1}{3}$ to his eldest daughter. How much of his estate is left to be divided among his other heirs?

Ans. \$756$\frac{2}{3}$.

13. John, Mary, and Charles have 8 cents between them. John and Charles have 5 cents, and Mary and Charles have 7 cents. How many cents has each of them? Ans. 1, 3, 4.

14. There are three flocks of sheep. The first contains 230, the second 110 more than the first, and the third 85 more than the second. How many in all? Ans. 995.

15. A man, born in 1789, died at the age of 46; his son died 27 years afterward and his daughter 13 years after his son. In what year did the daughter die?

16. The Duke of Wellington died in the year 1852, aged 83 years. Napoleon was born the same year as the Duke, and died in 1821. What was Napoleon's age?

17. A and B are traveling the same way, but they are 1mi., 4fur., 20 rods apart. B gains 8 rods upon A in every 25 rods. How far must B travel to overtake A?

Ans. $1{,}562\frac{1}{2}$ miles.

18. A man travels 100 miles by rail and 100 miles by stage; his average of travel was 16 miles per hour, and his rate by rail was 40 miles per hour. What was his rate per hour by stage? ANS. 10 miles.

19. A person bought a plank containing 76 square feet, which was 1ft., 7in. wide. How long was the plank?
ANS. 48 feet.

20. The fore-wheel of a carriage makes 8,400 revolutions in a journey. It is 6ft., 8in. around it, and five feet less in circumference than the hind-wheel. How many revolutions does the hind-wheel make? ANS. 4,800.

21. Find the product of the sum and difference of 25 and 16.

22. Divide the difference between 1,296 and 441 by the sum of 36 and 21, and give the quotient.

23. What are the prime factors of 9,800?

24. Find the greatest common divisor of 2,290 and 458.

25. Find the greatest common divisor of 1,435, 1,085, and 2,135.

26. Find the least common multiple of 2, 3, 4, 5, and 6; and also of 15, 18, 24, 40, and 50.

27. Subtract $\frac{1}{3}$ of $4\frac{1}{2}$ from $\frac{2}{7}$ of $9\frac{1}{4}$.

28. Multiply $3.31+4.06$ by $8.13-3.43$.

29. Divide $3.8+2.05$ by $8.6-3\frac{3}{4}$.

30. A man bought a horse and carriage. The horse cost $\frac{3}{5}$ as much as the carriage, and both together cost \$640. What was the cost of each? ANS. \$240, horse.

31. At a certain election the successful candidate had a majority of 120, which was $\frac{1}{9}$ of all the votes cast for himself. How many votes were cast in all? Ans. 2,040.

32. Divide \$357 between A, B, and C, so that B shall have $2\frac{1}{2}$ times as much as A, and C as much as A and B together. Ans. A, \$51; B, \$$127\frac{1}{2}$; C, \$$178\frac{1}{2}$.

33. A can do a piece of work in 3 days, B in 4 days, and C in 5 days. How long will it take them to do it by working all together? Ans. $1\frac{13}{47}$ days.

34. If 3,000 copies of a book of 11 sheets require 66 reams of paper, how much paper will 5,000 copies of a book of $12\frac{1}{2}$ sheets require? Ans. 125 reams.

35. $\frac{8}{11}$ of a certain number exceeds $\frac{4}{7}$ of the same by 156. What is the number? Ans. 1,001.

36. When a man was married he was 3 times as old as his wife; but fifteen years afterward he was only twice as old. What was his age when he married? Ans. 45 years.

37. If 20 men, 40 women, and 50 children receive £350 for 7 weeks' work, and 2 men receive as much as 3 women, or 5 children, what can a woman earn in a week? Ans. 10s.

38. A and B set out from the same place and in the same direction at the same time. A traveled 16 miles a day, and after traveling $7\frac{1}{2}$ days, turned and went back as far as B had traveled in $7\frac{1}{2}$ days. He then turned again, and pursuing his journey, overtook B 25 days after they first set out. How far did B travel each day? Ans. 10 miles.

Solution: B travels in $7\frac{1}{2}$ hours $\frac{7\frac{1}{2}}{25}$, or $\frac{3}{10}$ of the whole distance he travels; and A travels back this $\frac{3}{10}$ and *over it again*, in addition to going over the whole distance *once*, which really makes him travel $\frac{16}{10}$ of the distance from where they start to where they come together. But A travels 16×25 miles $= 400$ miles. 400 then is $\frac{16}{10}$ of said distance; and $250 = \frac{10}{10}$, or the distance. B's $\frac{3}{10}$ in $7\frac{1}{2}$ hours will then be 75 miles. $75 \div 7\frac{1}{2} = 10$, the number of miles B goes per day.

39. A merchant carried on business three years. The first year he gained a sum equal to $\frac{4}{11}$ of his original capital; the second year he lost $\frac{1}{4}$ of what he had at the end of the first year; the third year he gained $\frac{3}{10}$ of what he had at the end of the second year. He then had $14,625. How much had he gained in the three years? Ans. $3,625.

40. A house, rented for $12 a month, pays $7\frac{1}{2}\%$ interest. What is its value? Ans. $1,920.

41. The population of New York city in 1870 was 926,341, which was an increase of 14.6% in 10 years. What was its population in 1860? Ans. 808,325.

42. A can perform a piece of work in 9 days; A and B together can do the same in 6 days; A, B, and C working together can do it in four days. In what time can B and C working together perform the whole? Ans. $7\frac{1}{5}$ days.

43. If a certain number be increased by $1\frac{3}{4}$, this sum be diminished by $\frac{3}{8}$, this remainder multiplied by $5\frac{2}{5}$, and this product divided by $1\frac{2}{7}$, the quotient will be $7\frac{1}{2}$. What is the number? Ans. $\frac{23}{56}$.

44. A man killed an ox, which, when dressed, weighed $1.178\frac{3}{5}$ pounds; he kept $\frac{3}{5}$ of it for his own use and sold $\frac{2}{3}$ of the remainder. What was the value of the remainder at $11\frac{1}{4}$cts. a pound? Ans. $35.352.

45. If $3\frac{1}{4}$yds. of cloth $1\frac{3}{5}$yds. wide cost $\$3.46\frac{2}{3}$, what will be the cost of $42\frac{1}{4}$yds. $1\frac{1}{2}$yds. wide? Ans. $42.25.

46. A man bought $\frac{3}{8}$ of a vessel and sold $\frac{4}{5}$ of his share for $11,700, which was 30% above cost. What was the cost of the vessel? ANS. $30,000.

47. A merchant bought a piece of merino containing 32 yards for $25.60, and then marked it so that he could discount the asking price 4% and yet make 20% on cost. What did he mark it per yard? ANS. $1.00.

48. A man bought $43\frac{3}{4}$ yards of carpeting for $$58\frac{1}{3}$; he sold $\frac{2}{5}$ of the piece, gaining $$0.16\frac{2}{3}$ on each yard sold. How much did he receive for it? ANS. $26.25.

49. A and B traded with equal sums of money. A gained a sum equal to $\frac{1}{5}$ of his stock. B lost $220 and then had $\frac{1}{2}$ as much as A. What was the original stock? ANS. $550.

50. As A's money is 50% more than B's, how many per cent. less is B's than A's? ANS. $33\frac{1}{3}$%.

51. A man after spending $40 more than $\frac{3}{5}$ of his money had $60 less than $\frac{3}{7}$ of it left. How much money had he at first? ANS. $700.

52. A can do $\frac{1}{3}$ of a piece of work in 4 days, B can do $\frac{1}{4}$ of it in 5 days, C can do $\frac{1}{5}$ of it in 3 days, and D can do $\frac{1}{6}$ of it in $1\frac{1}{2}$ days. How long will it take all to do it together? ANS. $3\frac{3}{14}$ days.

53. A mast stands $\frac{1}{5}$ in the ground, $\frac{1}{4}$ in the water, and 33 feet in the air. How long is the mast? ANS. 60 feet.

54. What must I pay for Government 6 per cents. that my investment may yield $7\frac{1}{2}$%? ANS. 80%.

55. The longitude of Boston is 71°, $3\frac{1}{2}'$ W., and San Francisco 122°, 46′, 48″ W. What is the difference of time? and if it is 9 o'clock A.M. at Boston, what time is it at San Francisco? ANS. Difference in time, 5h., 26′, $53\frac{1}{5}''$.

56. A manufacturer employed men, women, and children. He had 3 women to every 2 men, and 3 children to every 2 women. To the men he paid \$1, to the women 50cts., and to the children 25cts. a day. At the end of 6 days he paid them all \$222. How many men did he employ?

ANS. 16 men.

57. A and B together can do a job in 7 days; but it would take A alone 12 days to do it. How long would it take B alone to do it? ANS. $16\frac{4}{5}$ days.

58. A merchant insures \$4,000 worth of silk at $2\frac{1}{2}\%$. What must be the face of his insurance that he may lose nothing in case of its destruction? ANS. \$4,102$\frac{22}{39}$.

59. To lose 25% on damaged goods how shall I mark those that cost me 16cts., 36cts., \$1.25, \$1.80, and \$6.56$\frac{1}{2}$?

60. A man dying left $33\frac{1}{3}\%$ of his property to his wife, 50% of the remainder to his son, 75% of the residue to his daughter, and the balance, \$120, to a faithful servant. How much did each receive?

61. If a man borrow \$10,000 in Virginia at 6%, and lends it in Alabama at 8%, how much does he gain per year?

ANS. \$200.

62. What sum must I invest in United States 6's, selling at $2\frac{1}{2}\%$ premium, to secure an annual income of \$840?

SOLUTION: $\frac{840 \times .02\frac{1}{2}}{.06}$ = ANS. \$14,350.

63. What is the rate of income upon money invested in 6% bonds purchased at a discount of 10%? ANS. $6\frac{2}{3}\%$.

64. A had $\frac{2}{5}$ of $\frac{5}{6}$ of $3\frac{3}{4}$ times \$31,448, and paid $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{7}{10}$ of it for a farm. How much money had he remaining? ANS. \$35,379.

65. What is the difference between the simple, annual, and compound interest of \$700 at 5% per annum for 3 years?
ANS. Simple, \$105; annual, \$110.25; compound, \$110.34.

66. At what rate per annum, simple interest, will any sum of money double itself in 3, 4, 5, 6, and 10 years respectively? ANS. $33\frac{1}{3}$%, 25%, 20%, $16\frac{2}{3}$%, 10%.

67. At what rate per annum, simple interest, will any sum triple itself in 3, 5, 6, 11, and 20 years respectively?
ANS. $66\frac{2}{3}$%, 40%, $33\frac{1}{3}$%, $18\frac{2}{11}$%, 10%.

68. Bought a hogshead of molasses for a certain sum, but 16 gallons having leaked out, the remainder was sold at \$1.87½ per gallon, at a loss of 6% on cost. How much was the cost? ANS. \$93.75.

69. A merchant bought 400 yards broadcloth at \$3.00 per yard. He sold A 60 yards at \$3.50, B 125 yards at \$3.40, C 75 yards at \$3.20, and the balance to D at \$2.75. What per cent. profit did he make on the whole? ANS. 5%.

70. How much ready money will now pay a debt of \$1,342.50, due 125 days hence, at $6\frac{1}{2}$%? ANS. \$1,313.26.

71. A man agrees to execute a contract in 60 days, and places 30 men on the work. At the end of 48 days the job is but half completed. How many men must he employ the rest of the time to fulfil his contract? ANS. 120.

72. How many bushels of barley at \$0.80 in gold can be bought for \$385.56 in currency, when gold is $118\frac{1}{8}$%?
ANS. 408 bushels.

73. A path is laid out in a rectangular garden, whose length is 105 yards and whose breadth is 95 yards. The path which runs completely around is 3 feet wide, and its outer edge is four feet from the wall. How many square feet does the path contain? ANS. 3,468.

74. A lady wishes to carpet a floor 15ft., 9in. wide, and 22ft., 6in. long, with carpeting $\frac{3}{4}$yd. wide. How many yards of carpet? and what will it cost at $2.50 a yard?

ANS. $131.25.

75. A person proposes to sell his horse by lottery. If the tickets be $3 he will lose $20; but if they are $4 each he will gain $20. What was the value of his horse? and what was the number of his tickets?

ANS. $140, and 40 tickets.

76. A surveyor, measuring a piece of land in the form of a rectangle, found one side to be 55 chains and 50 links, and the other 63 chains, 24 links. How many acres did the farm contain? ANS. 350 acres, 3 rods, 37.12 poles.

77. A merchant having sold a number of yards from a piece of cloth found there were 8.75 yards left, which was 65% less than the quantity cut off. How many yards in the piece at first? ANS. 33.75 yards.

78. A house that cost $15,725 rents for $1,478.15; the insurance is $\frac{4}{5}$%, and the repairs $\frac{6}{10}$%, each year. What rate of interest does it pay? ANS. 8%.

79. A and B traded in company. A put in $845 for 300 days and received $\frac{3}{8}$ of the gain. The number of dollars B put in was equal to the number of days it was used in trade. What was B's capital? ANS. $650 for 650 days.

80. A man bought $\frac{3}{8}$ of a vessel, and sold $\frac{4}{5}$ of his share for $11,700, which was 30% above cost. What was the cost of the vessel? ANS. $30,000.

81. If $3\frac{1}{4}$ yards of cloth $1\frac{3}{5}$ yards wide cost 3.46\frac{2}{3}$, what will be the cost of $42\frac{1}{4}$ yards $1\frac{1}{2}$ yards wide? ANS. $42.25.

82. A man pays $6 yearly for tobacco from the age of 16 until he is 60, when he dies leaving to his heirs $500.

What might he have left if he had dispensed with this useless weed and loaned the money paid for it at the end of each year at 6% compound interest? Ans. $1,692.56.

83. How many feet of boards will it take to cover a barn $40\frac{1}{2}$ feet long, 30 feet, 10 inches wide, 18 feet, 4 inches high, and each side of the roof 16 feet, allowing 385 feet, 3 inches for the gables? Ans. 4,296ft., 7in.

84. A, B, and C have business transactions together, whereby they gain $18,049.60. A furnished $22,000 for 12 months, B $18,600 for 10 months, and C 30,000 for 7 months. To what part of the gain is each entitled?

Ans. A, $7,219.84; B, $5,086.70$\frac{6}{11}$; C, $5,743.05$\frac{5}{11}$.

85. A person being asked the hour of the day answered that the time past noon was $\frac{4}{5}$ of the time to midnight. What was the time? Ans. 5 o'clock, 20m.

86. If a quantity of bread will last 1,500 men for 12 weeks at the rate of 20 ounces per day for each man, how long will the same bread last 2,500 men at the rate of 16 ounces per day for each man? Ans. 9 weeks.

87. A bankrupt's assets amounted to $4,000. To A he owed $\frac{1}{2}$ of the assets, to B $\frac{1}{3}$ of the assets, to C $\frac{1}{5}$ of the assets, and to D $\frac{1}{6}$ of the assets. What part of the assets did A receive? Ans. $\frac{15}{36}$, or $1,666.66$\frac{2}{3}$.

88. The head of a fish is 9 inches long; its tail was as long as its head and half its body; and its body as long as its tail and head together. How long was the fish?

Ans. 72 inches.

89. If a pound of tea be worth 0.62\frac{1}{2}$ what is .8 of a pound worth? Ans. 50 cents.

90. Sold a horse at $33\frac{1}{3}\%$ gain, and with the money bought another horse, which I sold for \$120 and lost 25%. Did I gain or lose by the trade, and how much?

ANS. Neither.

91. A man wishing to sell a horse asked 25% more than cost. He finally sold it 15% less than his asking price and gained \$7.50. How much did the horse cost him, and what was his asking price? ANS. Cost \$120; Asking price, \$150.

92. Bought a horse for \$4,500, and paid \$500 in cash and the balance in eight equal annual instalments. What is the mean time for paying the balance? ANS. $4\frac{1}{2}$ years.

93. A merchant insures a ship and cargo for \$79.325 at $4\frac{3}{8}\%$; the policy covering both the property and premium. What is the value of the ship and cargo? ANS. \$76,000.

94. The horizontal distance between the eaves of a certain building is 40 feet, the elevation of the ridge above the eaves is 15 feet. What is the length of the rafters? ANS. 25 feet.

95. What sum of money with its semi-annual dividends of 4% invested with it will amount to \$25,000 in 3 years?

ANS. \$19,758.16.

DIRECTIONS: Get the amount of \$1 on interest, 6 periods, at 4%.

96. A father left \$10,000 to his two sons, aged respectively 14 and 18, to be divided between them so that the shares at simple interest at 5% should be the same when each was 21 years old. What was the share of each?

ANS. \$5,400 and \$4,600.

97. What sum is that from which if you take $\frac{2}{7} \times \frac{3}{8}$, and to the remainder add $\frac{7}{16} \times \frac{1}{20}$, the result will be 10?

ANS. $10\frac{191}{2240}$.

98. When gold is quoted at 162% what is the value in gold of a United States note of the denomination of $10?

ANS. $6.17.

99. A. Bristow received a legacy of $3,000, $800 of which was payable in 9 months, $800 in one year, and the balance in two years. What would be the present payment, discounting at 7%?

ANS. $2,735.83.

100. There is a room 20 feet long, 12 feet wide, and 12 feet high. What is the distance from the upper corner to the lower opposite diagonal corner? If a small parallelopiped stands closely in one corner of said room, whose length is 20 inches, its width 12 inches, and its height 12 inches, what is the distance from the same upper corner of the room to the corresponding upper corner of the block in the lower opposite corner of the room?

ANS. $23\frac{3}{100}$+ and $21\frac{92}{100}$+ feet.