

THE SLIDE RULE

A Complete Manual

Alfred L. Slater

HOLT | RINEHART | WINSTON

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ALFRED L. SLATER

Los Angeles City College



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PREFACE

The purpose of this manual is to provide the student with a large number of illustrative examples together with a plentiful supply of practice problems. Answers are given for all exercises. Separate exercise sets feature "formula-type" expressions, many of which require intermediate additions or subtractions. These are representative of computations the student will often encounter in practice.

The first twelve chapters cover the C, D, CI, CF, DF, CIF, A, B, and K scales. The double-length scale (R, Sq, $\sqrt{\quad}$) is also discussed, and alternate procedures are presented utilizing this scale in place of the A-B scales. No knowledge of trigonometry or logarithms is assumed for this portion of the manual.

The remainder of the text covers the trigonometric scales (S, ST, T), the log scale (L), and the log-log scales (LL). The base-10 LL scales and the hyperbolic scales (Sh, Th) are briefly treated in the appendix.

The presentation is consistent throughout. Each new slide rule technique is introduced by examples which are carefully broken down step-by-step so that the student should have no difficulty following on his own slide rule. These are followed immediately by several similar drill problems with answers displayed. Thus, the student may test his understanding at once before getting to the main exercise set for the section. Normally, a good deal of classroom time is given to mastering the basic operational scales: C, D, CI, CF, DF, and CIF. Beyond this, time considerations will determine the selection of additional topics for class

discussion. It is hoped that the text has sufficient clarity and detail to encourage a certain amount of independent learning by the student.

All of the material in this manual has been used in syllabus form for the past several years at Los Angeles City College. During this period, corrections and suggestions by the mathematics faculty at the college have been gratefully received by the author.

*Los Angeles, California
February 1967*

A.L.S.

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Chapter 1

INTRODUCTION: LOCATION OF DECIMAL POINT

1.1 Introduction

The slide rule is used principally to perform multiplication, division, and combinations of these operations. Also, it is very easy to find squares, cubes, square roots, and cube roots of numbers. Once you have learned to use the slide rule, you will be able to quickly find answers to the following:

$$463 \times 1.67 \times .0277 = ?$$

$$\frac{7.64 \times 5.22}{(3.62)^2} = ?$$

$$\sqrt{516} = ?$$

$$\sqrt[3]{.0744} = ?$$

$$\frac{175}{2.8} \sqrt{\frac{4.6 \times 17.2}{3.77}} = ?$$

All standard rules also have scales enabling you to find trigonometric[†] functions and common logarithms; furthermore, if your rule has “log log” scales you can raise numbers to any power. Knowledge of these scales will enable you to compute the following:

$$14.2 \sin 46.5^\circ = ?$$

$$\log 45.7 = ?$$

$$23^{1.4} = ?$$

$$\sqrt[5]{33.2} = ?$$

This first chapter is concerned only with the problem of placing the decimal point. With Chapter 2 you start using the slide rule and it is important that, beginning with this chapter, you read the manual with your slide rule always at hand. The illustrative examples are carefully broken down step-by-step, and you should reproduce each move on your slide rule as it is described. Be sure you understand thoroughly what each move accomplishes. There are many drill exercises (with answers) for you to practice on.

Mastery of the slide rule is a skill that can only be acquired through repetitive drill; only in this way will you reach the point where you automatically perform the operations quickly and accurately.

1.2 Slide rule does not ordinarily locate the decimal point

Aside from certain operations involving the L and LL scales, the slide rule does not indicate the position of the decimal point in the answer; it merely gives the principal digits, and you must place the decimal point yourself. Thus, if the slide rule is used to multiply 2.52 by 3.07, the answer will simply be read as a number whose digits are "seven-seven-four." Clearly, in this case, the answer must be 7.74. Again, suppose the problem is to divide 64.3 by 2.17. The slide rule will indicate the answer to be a number whose digits are "two-nine-six." Here, it is evident that the answer must be 29.6.

1.3 Locating the decimal point by inspection

In many slide rule operations, the decimal point may be placed simply by inspection. This is illustrated by the following examples:

Example 1: $3.96 \times 12.45 = \text{"493"}$

By "493" we mean the slide rule reading with decimal point *unplaced*; this notation will be used throughout the text. In this example, we think of the product as approximately 4 times 12; hence, answer must be **49.3**.

Example 2: $\frac{54.3}{2.78} = \text{"1952"}$

Here, we are roughly dividing 54 by 3, so that quotient should be about 18. Thus, answer must be **19.52**.

Example 3: $\frac{.00783}{2.16} = \text{"363"}$

In this case, the answer is approximately .008 divided by 2, or about .004. Therefore, answer must be **.00363**.

Example 4: $3.75 \times .0188 = \text{"705"}$

We are roughly multiplying 4 by .02, so that product should be about .08. It follows that the result must be **.0705**.

To give you some practice in doing this, the following exercise contains a number of simple products and quotients with the slide rule answers indicated; you are required to locate the decimal point properly.

Exercise 1-1

Locate the decimal point in the indicated slide rule answer:

- | | |
|---|---|
| 1. $2.8 \times 4.2 = \text{"1176"}$ | 17. $\frac{56.2}{3.15} = \text{"1785"}$ |
| 2. $5.6 \times 6.2 = \text{"347"}$ | 18. $\frac{.0776}{2.66} = \text{"292"}$ |
| 3. $26 \times 3.7 = \text{"962"}$ | 19. $\frac{87.5}{2.39} = \text{"366"}$ |
| 4. $68 \times 0.26 = \text{"1768"}$ | 20. $\frac{114.5}{2.56} = \text{"447"}$ |
| 5. $15.2 \times 0.47 = \text{"714"}$ | 21. $48.8 \times 0.283 = \text{"1381"}$ |
| 6. $165 \times 0.74 = \text{"1221"}$ | 22. $36.0 \times 3.24 = \text{"1166"}$ |
| 7. $\frac{14}{2.6} = \text{"538"}$ | 23. $\frac{107.5}{45.2} = \text{"238"}$ |
| 8. $\frac{36}{5.7} = \text{"632"}$ | 24. $547 \times 0.640 = \text{"351"}$ |
| 9. $\frac{75}{27} = \text{"278"}$ | 25. $\frac{54}{77} = \text{"702"}$ |
| 10. $\frac{340}{7.2} = \text{"472"}$ | 26. $2630 \times 0.533 = \text{"1400"}$ |
| 11. $1.75 \times 4.24 = \text{"742"}$ | 27. $\frac{830}{5.25} = \text{"1581"}$ |
| 12. $3.22 \times 15.7 = \text{"505"}$ | 28. $5050 \times 1.85 = \text{"935"}$ |
| 13. $26.2 \times 0.485 = \text{"1270"}$ | 29. $\frac{4.75}{52.7} = \text{"902"}$ |
| 14. $25.8 \times 1.77 = \text{"457"}$ | 30. $25.9 \times 4.40 = \text{"1140"}$ |
| 15. $46.6 \times 2.07 = \text{"966"}$ | 31. $42.6 \times 18.7 = \text{"797"}$ |
| 16. $3.79 \times 5.20 = \text{"1970"}$ | |

32. $.0366 \times 7.23 = \text{"265"}$

37. $\frac{1}{3.22} = \text{"311"}$

33. $\frac{12.66}{27.3} = \text{"464"}$

38. $\frac{148.5}{0.92} = \text{"1615"}$

34. $\frac{.00427}{8.15} = \text{"524"}$

39. $43.6 \times 21.7 = \text{"946"}$

36. $\frac{243}{52.7} = \text{"461"}$

40. $\frac{.000862}{1.84} = \text{"468"}$

1.4 Shifting decimal point in products and quotients

In a *product* of two numbers, if the decimal point of one of the numbers is moved a certain number of places in one direction, the decimal point of the other number must be moved the same number of places in the *opposite* direction. Ordinarily, the object is to make one of the factors a number between 1 and 10; it then becomes easier to place the decimal point in the answer.

Example 1: $1365 \times .0000554 = \text{"755"}$

Here, we may shift the decimal point in the first factor three places to the left, thus making it a number between 1 and 10. We must then shift the decimal point three places to the right in the second factor; this corresponds to dividing and multiplying by 1000. Thus, the product becomes:

$$\begin{array}{c} \uparrow \quad \uparrow \quad \uparrow \\ 1365 \times .0000554 = 1.365 \times .0554 = \text{"755"} \end{array}$$

It is now clear that the answer must be **.0755**.

Example 2: $304 \times 1870 = \text{"568"}$

In this case, we may shift the decimal point two places to the left in the first factor, and two places to the right in the second factor:

$$\begin{array}{c} \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ 304 \times 1870.00 = 3.04 \times 187,000 = \text{"568"} \end{array}$$

It is now evident that the answer must be **568,000**.

In a *quotient* of two numbers, if the decimal point in the numerator is moved a certain number of places in one direction, the decimal point in the denominator must be moved the same number of places in the *same* direction. Here, the object is usually to make the denominator a number between 1 and 10, thus simplifying the placement of the decimal point.

Example 3: $\frac{.0713}{.000225} = \text{"317"}$

We shift the decimal point four places to the right in both numerator and denominator:

$$\frac{\boxed{.0713} \downarrow}{\boxed{.000225} \uparrow} = \frac{713}{2.25} = \text{"317"}$$

Note that this corresponds to multiplying numerator and denominator by 10,000. Clearly, the answer must be **317**.

Example 4: $\frac{1}{4360} = \text{"229"}$

Shift decimal point three places to the left in both numerator and denominator:

$$\frac{\boxed{.001} \downarrow}{\boxed{4360} \uparrow} = \frac{.001}{4.36} = \text{"229."} \quad \text{Answer must be } \mathbf{.000229}.$$

Example 5: $\frac{32.1}{.00643} = \text{"500"}$

Shift decimal point three places to the right in both numerator and denominator:

$$\frac{\boxed{32.100} \downarrow}{\boxed{.00643} \uparrow} = \frac{32,100}{6.43} = \text{"500."} \quad \text{Answer must be } \mathbf{5000}.$$

Exercise 1-2

Locate the decimal point in the indicated slide rule answers:

1. $2650 \times .000165 = \text{"437"}$

7. $\frac{.000724}{.026} = \text{"279"}$

2. $324 \times 205 = \text{"664"}$

8. $\frac{1.07}{4650} = \text{"230"}$

3. $.0642 \times .0322 = \text{"207"}$

9. $\frac{27.6}{.0032} = \text{"862"}$

4. $.000724 \times 217 = \text{"1570"}$

5. $.026 \times 1640 = \text{"426"}$

6. $\frac{361}{.0742} = \text{"486"}$

10. $\frac{12,750}{327} = \text{"390"}$

11. $.00724 \times .036 = \text{"260"}$

12. $46,000 \times .0023 = \text{"1059"}$

13. $\frac{2.75}{3750} = \text{"733"}$

14. $\frac{0.622}{.00045} = \text{"1382"}$

15. $0.43 \times .00026 = \text{"1118"}$

16. $\frac{1}{.0073} = \text{"1370"}$

17. $\frac{1}{3750} = \text{"267"}$

18. $565 \times 407 = \text{"230"}$

19. $.0064 \times .0024 = \text{"1538"}$

20. $\frac{5640}{23.6} = \text{"239"}$

21. $\frac{.0043}{.075} = \text{"573"}$

22. $\frac{12.6}{.00165} = \text{"763"}$

23. $43,000 \times .00611 = \text{"262"}$

24. $265 \times 720 = \text{"1908"}$

25. $.0045 \times .00133 = \text{"598"}$

26. $\frac{1}{.000505} = \text{"1980"}$

27. $\frac{1}{3700} = \text{"270"}$

28. $\frac{14.6}{.000327} = \text{"447"}$

29. $\frac{.0764}{.000223} = \text{"343"}$

30. $420 \times .00065 = \text{"273"}$

1.5 Expressions involving several numbers

In many cases involving more than two numbers, the decimal point may still be located by inspection. However, if the expression appears at all complicated, it is safer to rewrite it with the numbers rounded off to one significant digit.

Example 1: $\frac{37.6 \times 23.8}{71.6} = \text{"1250"}$

Rounding off:

$$\frac{37.6 \times 23.8}{71.6} \approx \frac{40 \times 20}{70} = \frac{80}{7} \approx 11. \quad \text{Answer must be } \mathbf{12.50}.$$

Note: The "wavy" equal sign means "approximately equal to."

Example 2: $\frac{236 \times 61.4}{18.85 \times 4.45} = \text{"1727"}$

Rounding off:

$$\frac{236 \times 61.4}{18.85 \times 4.45} \approx \frac{200 \times 60}{20 \times 4} = 150. \quad \text{Answer must be } \mathbf{172.7}.$$

Example 3: $\frac{92.3}{2.37 \times 213} = \text{"1829"}$

Rounding off:

$$\frac{92.3}{2.37 \times 213} \approx \frac{90}{2 \times 200} = \frac{90}{400} = \frac{9}{40} \approx 0.2.$$

Answer must be **0.1829**.

Example 4: $\frac{\pi \times 17.4 \times 2.75}{.0584} = \text{"257"}$

Rounding off:

$$\frac{\pi \times 17.4 \times 2.75}{.0584} \approx \frac{3 \times 20 \times 3}{.06} = \frac{180}{.06} = \frac{18,000}{6} = 3000.$$

Clearly, answer must be **2570**.

Example 5: $\frac{.00332 \times 14.7}{.00072} = \text{"678"}$

Shifting decimal points and rounding off:

$$\frac{\begin{array}{c} \downarrow \\ .00332 \times 14.7 \\ \uparrow \\ .00072 \end{array}}{\quad} = \frac{33.2 \times 14.7}{7.2} \approx \frac{30 \times 15}{7} = \frac{450}{7} \approx 60.$$

Answer must be **67.8**.

Exercise 1-3

Locate the decimal point in the indicated slide rule answers:

1. $\frac{65.2 \times 71.6}{107} = \text{"436"}$

3. $\frac{26 \times 390}{4.2 \times 5} = \text{"483"}$

2. $\frac{11.45 \times 243}{25.6} = \text{"1087"}$

4. $\frac{2.61 \times 14.1}{6.05} = \text{"608"}$

5. $\frac{147}{3.45 \times 4.26} = \text{"1000"}$
6. $\frac{362 \times .072}{645} = \text{"404"}$
7. $\frac{32.4 \times 1.62}{11.4} = \text{"460"}$
8. $\frac{37.4 \times 5.63}{9.22} = \text{"228"}$
9. $\frac{148}{3.6 \times 4.12} = \text{"997"}$
10. $\frac{\pi \times 21.2}{15.4} = \text{"433"}$
11. $\frac{4600}{48.2 \times 2.06} = \text{"463"}$
12. $\frac{5.72 \times 43.2}{2.40 \times 3.72} = \text{"277"}$
13. $\frac{16.2 \times 27.2}{5.10 \times 5.90} = \text{"1465"}$
14. $\frac{236 \times 5.60}{8.45 \times \pi} = \text{"498"}$
15. $\frac{278 \times 562}{17.2 \times 8.41} = \text{"1078"}$
16. $\frac{1750 \times 43.6}{36.3 \times 7.22} = \text{"291"}$
17. $\frac{49.2 \times 576}{7250 \times \pi} = \text{"1245"}$
18. $\frac{.0043 \times 56.2}{30.4} = \text{"795"}$
19. $\frac{7850 \times .00034}{.00072} = \text{"371"}$
20. $\frac{23 \times 356 \times 75}{12 \times 46} = \text{"1113"}$
21. $\frac{582 \times 17 \times 62}{432 \times 108} = \text{"1315"}$
22. $.0042 \times 563 \times 27 \times 7.6 = \text{"485"}$
23. $0.46 \times 72.3 \times 5.22 = \text{"1735"}$
24. $\frac{562,000 \times .00463}{2.77} = \text{"940"}$
25. $\frac{.0475 \times 3460}{6.44} = \text{"255"}$
26. $\frac{5.72 \times 4.65}{.00324} = \text{"821"}$
27. $\frac{463 \times .00524}{3.22} = \text{"753"}$
28. $\frac{245}{.000344 \times 6340} = \text{"1124"}$
29. $\frac{415 \times 625}{765,000} = \text{"339"}$
30. $\frac{3750 \times .00728}{.0263 \times 185} = \text{"561"}$
31. $\frac{.00372 \times 15.6}{.00086} = \text{"674"}$
32. $240 \times 36 \times .0057 \times 0.43 = \text{"212"}$
33. $\frac{37.2 \times .072 \times 6.3}{256 \times .0041} = \text{"1608"}$
34. $\frac{382 \times 624}{21 \times 43 \times 12} = \text{"220"}$
35. $\frac{.036 \times .0073}{.0052} = \text{"505"}$
36. $\frac{1}{.072 \times 6.4 \times 0.32} = \text{"679"}$
37. $\frac{.00468 \times 6250}{3.81} = \text{"767"}$
38. $\frac{2170}{25 \times 5.64 \times 3.44} = \text{"447"}$
39. $\frac{.0032 \times .0843 \times 164}{21 \times 0.63} = \text{"334"}$
40. $146 \times 21.6 \times .0072 = \text{"227"}$

1.6 Scientific notation

A number is written in scientific notation when it is written in the form $M \times 10^n$, where M is a number between 1 and 10, and n is a positive or negative integer.

Example 1: $4630 = 4.63 \times 1000 = 4.63 \times 10^3$

Example 2: $.000706 = 7.06 \div 10,000 = 7.06 \times 10^{-4}$

It is clear that, in each case, the decimal point has been shifted to give a number between 1 and 10; furthermore, the number of places it has been shifted determines the exponent of the power of 10. If it has been moved to the left, the exponent is positive; if to the right, the exponent is negative.

Other examples of numbers converted to scientific notation follow:

<i>number</i>	<i>exponent</i>	<i>number in scientific notation</i>
5,740,000	6	5.74×10^6
.0000307	-5	3.07×10^{-5}
0.624	-1	6.24×10^{-1}
4.75	0	$4.75 \times 10^0 = 4.75$

1.7 Laws of exponents

In the next section we make use of the following laws of exponents:

1. When powers of the same base are *multiplied*, the exponents are *added*.
2. When powers of the same base are *divided*, the exponents are *subtracted*.

Examples:

a. $10^3 \times 10^{-2} \times 10^4 = 10^{3-2+4} = 10^5$

b. $\frac{10^5}{10^3} = 10^{5-3} = 10^2$

c. $\frac{10^5 \times 10^{-3}}{10^2} = 10^{5-3-2} = 10^0 = 1$

d. $\frac{10^2 \times 10^{-3} \times 10^4}{10^{-2} \times 10^8} = 10^{2-3+4+2-8} = 10^{-3}$

1.8 Approximation using scientific notation

When rounding off numbers to one significant digit, it is often helpful to put them in scientific notation at the same time. The following examples illustrate the procedure:

Example 1: $\frac{.00713 \times 42,300}{324 \times .0000516} = \text{"1802"}$

Round off to single digits using scientific notation:

$$\frac{.00713 \times 42,300}{324 \times .0000516} \approx \frac{7 \times 10^{-3} \times 4 \times 10^4}{3 \times 10^2 \times 5 \times 10^{-5}}$$

Now, combining exponents:

$$\frac{7 \times 4}{3 \times 5} \times 10^{-3+4-2+5} = \frac{28}{15} \times 10^4 \approx 2 \times 10^4$$

Therefore, answer must be 1.802×10^4 or **18,020**.

The above operation may be shortened by simply writing the exponent above each factor of the numerator, and below each factor of the denominator:

$$\frac{\begin{matrix} (-3) & (4) \\ .00713 \times 42,300 \end{matrix}}{\begin{matrix} (2) & (-5) \\ 324 \times .0000516 \end{matrix}} \approx \frac{7 \times 4}{3 \times 5} \times 10^{-3+4-2+5} \approx 2 \times 10^4$$

Example 2: $523 \times 46.2 \times 73,100 = \text{"1763"}$

Approximate as follows:

$$\begin{matrix} (2) & (1) & (4) \\ 523 \times 46.2 \times 73,100 \end{matrix} \approx 5 \times 5 \times 7 \times 10^{2+1+4} = 175 \times 10^7 = 1.75 \times 10^9$$

Answer must be 1.763×10^9 .

Example 3: $\frac{.000273 \times 6.33 \times 6440}{0.821 \times 237,000} = \text{"572"}$

$$\frac{\begin{matrix} (-4) & (0) & (3) \\ .000273 \times 6.33 \times 6440 \end{matrix}}{\begin{matrix} (-1) & (5) \\ 0.821 \times 237,000 \end{matrix}} \approx \frac{3 \times 6 \times 6}{8 \times 2} \times 10^{-5} = \frac{27}{4} \times 10^{-5} \approx 7 \times 10^{-5}$$

Answer must be 5.72×10^{-5} or **.0000572**.

Example 4: $\frac{4.12}{275 \times 36.2 \times .0074} = \text{"559"}$

$$\frac{\begin{matrix} (0) \\ 4.12 \end{matrix}}{\begin{matrix} (2) & (1) & (-3) \\ 275 \times 36.2 \times .0074 \end{matrix}} \approx \frac{4}{3 \times 4 \times 7} \times 10^0 = \frac{1}{21} \times 10^0 \approx .05$$

Answer must be **.0559**.

Example 5: $\frac{21.2 \times .00465}{0.733 \times 1740 \times 10^{-5}} = \text{"773"}$

$$\frac{\overset{(1)}{21.2} \times \overset{(-3)}{.00465}}{\underset{(-1)}{0.733} \times \underset{(3)}{1740} \times \underset{(-5)}{10^{-5}}} \approx \frac{2 \times 5}{7 \times 2 \times 1} \times 10^1 = \frac{50}{7} \approx 7$$

Answer must be **7.73**.

Exercise 1-4

Locate the decimal point using scientific notation. If the answer is very small or very large, it is better to leave it in the scientific notation form.

1. $.000482 \times .0000612 = \text{"295"}$

2. $3760 \times 28 \times 4810 = \text{"506"}$

3. $\frac{.0000216 \times 587}{317,000} = \text{"400"}$

4. $\frac{2430 \times 0.416}{.000136} = \text{"744"}$

5. $\frac{.000742}{124,500} = \text{"596"}$

6. $.00720 \times 2410 \times 35,000 = \text{"607"}$

7. $\frac{572 \times 43,000}{.00375} = \text{"656"}$

8. $\frac{6.73 \times 0.217 \times 5430}{.000245 \times 38.2} = \text{"846"}$

9. $\frac{4.35 \times .001675}{5840 \times 21.6} = \text{"578"}$

10. $\frac{1}{68.2 \times .000781 \times 0.207} = \text{"907"}$

11. $\frac{100}{1.63 \times 4830 \times 56.7} = \text{"224"}$

12. $.00713 \times 560 \times 10^{-3} = \text{"400"}$

13. $\frac{1645 \times .0724}{36.1 \times 4310} = \text{"767"}$

14. $\frac{10^4}{25.2 \times .0756 \times 644} = \text{"815"}$

15. $176 \times 93.5 \times .000455 = \text{"749"}$

16. $\frac{6530 \times 10^5}{1726 \times 23.7 \times 408} = \text{"391"}$

17. $\frac{.0756 \times 545 \times 10^{-3}}{.00534 \times 8670 \times 17.5} = \text{"509"}$

18. $.00577 \times 368 \times .0305 = \text{"648"}$

19. $\frac{1}{38.9 \times .00219 \times 6.44} = \text{"1824"}$

20. $24.9 \times 1740 \times 522 \times 10^{-4} = \text{"2260"}$

21. $\frac{.00683 \times .0468}{.000792 \times .0502} = \text{"803"}$

22. $\frac{56.3 \times 16.4 \times 10^{10}}{.000745 \times 136.5} = \text{"907"}$

23. $\frac{.000713 \times .0644}{43.9 \times 1.4 \times 10^{-6}} = \text{"747"}$

24. $\frac{340 \times 56.2 \times 755 \times 12}{.00439} = \text{"394"}$

$$25. \frac{5.31 \times 66 \times 134 \times 10^{-4}}{.000294} = \text{"1598"}$$

$$26. \frac{1540 \times 36.2 \times .0426}{570} = \text{"416"}$$

$$27. \frac{.0774}{39.2 \times 143 \times 0.22} = \text{"627"}$$

$$28. 436 \times 124 \times 15.6 \times 56.4 = \text{"475"}$$

$$29. \frac{19,000 \times 43.6}{285 \times .0622} = \text{"467"}$$

$$30. \frac{3.41 \times 563 \times .0071}{22.4 \times 172,000} = \text{"354"}$$

Chapter 2

READING THE C AND D SCALES

2.1 Physical parts of the slide rule

That part of the slide rule which is fixed between the end plates is called the *body*; the long, movable center portion is called the *slide*, and the glass runner is called the *indicator*. The thin, vertical line on the indicator is referred to as the *hairline*. These parts are illustrated in Figure 2.1.

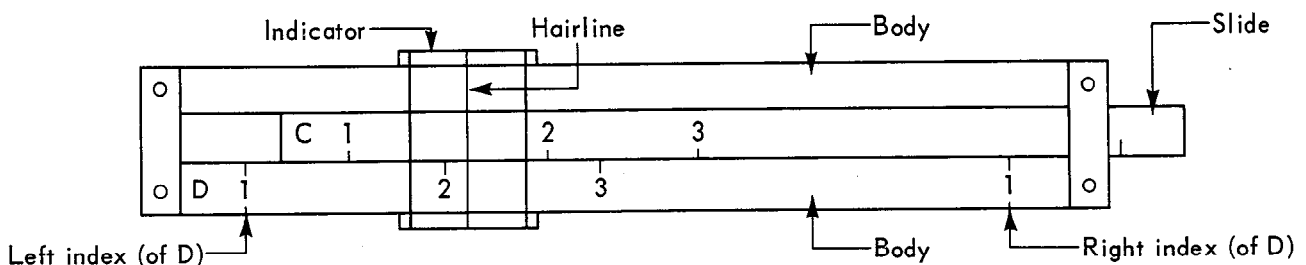


Figure 2.1

2.2 Uniform and nonuniform scales

An ordinary ruler or yardstick is an example of a uniform scale. Here, the distance, say, between the 2-inch mark and the 3-inch mark is the same as the distance between the 7-inch mark and the 8-inch mark; that is, the interval between consecutive inch marks is the same on all parts of the scale.

On a nonuniform scale, the distance between consecutive primary markings does not remain the same on all parts of the scale. Thus, the distance between 1 and 2 may be greater than the distance between 2 and 3, and so on. Now, if a conventional slide rule is examined, it will be seen that the L scale is the only uniform scale on the rule; all other scales are nonuniform.

2.3 Description of the C and D scales

The C and D scales are identical scales located on the slide and body respectively. You will note that these scales have ten primary marks which are numbered with the large numerals: 1, 2, 3, . . . 8, 9, 1. The primary mark corresponding to the large numeral 1 is called the index of the scale; hence, there are two indexes associated with each scale—a left index and a right index (see Figure 2.1).

Observe that the distance between the primary marks 1 and 2 is greater than the distance between 2 and 3, which, in turn, is greater than the distance between 3 and 4, etc. This nonuniform characteristic of the scale results from the fact that the scale distances are proportional to the logarithms of the corresponding numbers. Since the logarithm of 1 is zero, this explains why the scales start with the numeral 1.

Between the primary marks 1 and 2, there are ten secondary divisions which are numbered with the small numerals 1, 2, 3, . . . 8, 9. These secondary divisions are in turn divided into ten small intervals. Each of these smallest intervals may be taken to represent *one* unit.

Between the primary marks 2 and 3, there are also ten secondary divisions (not numbered), and each of these is in turn divided into five small intervals. Each of these smallest intervals then represents *two* units. The same is true for the portion of the rule between primary numbers 3 and 4.

Between 4 and 5, there are again ten secondary divisions, and each of these is divided into two small intervals; hence, each of these smallest intervals represents *five* units. The same holds true for the remaining primary divisions between 5 and the right index.

Portions of the D scale are shown in Figure 2.2, illustrating the markings.

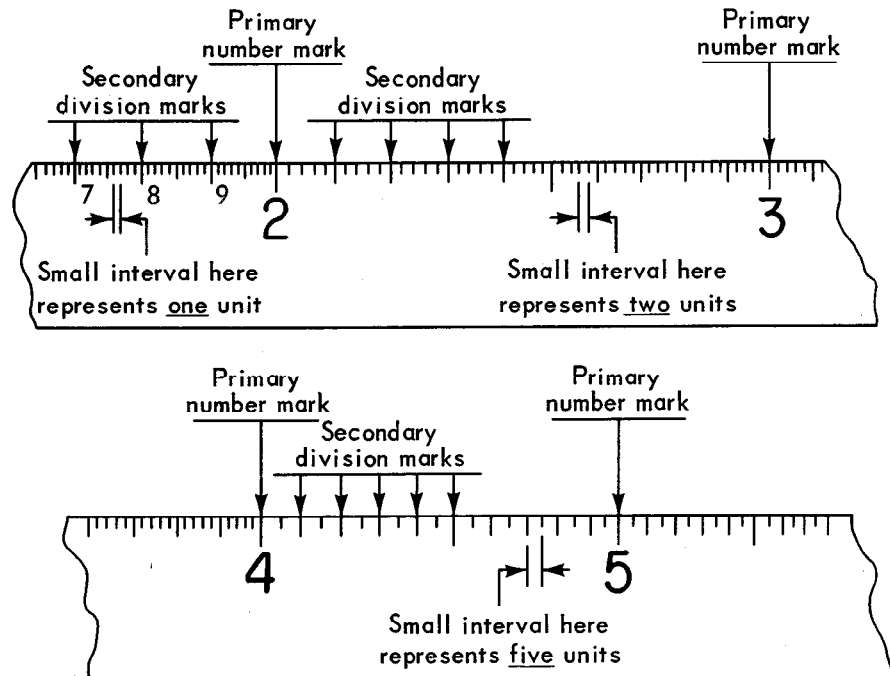


Figure 2.2

2.4 Scale setting independent of decimal point

As previously noted, the setting corresponding to a particular number is not affected by the position of the decimal point. Thus, the setting "237" may represent the numbers 23.7, 2.37, .00237, 2370; that is, the position on the scale is determined only by the digits "two-three-seven."

2.5 Reading the C and D scales

Consider that portion of the D (or C) scale between the primary numbers 3 and 4 (see Figure 2.3). Now if the hairline is positioned directly over the primary number 3, the setting corresponds to "300." This setting may represent the numbers 30, or .0300, or 30,000, and so on. If the hairline is moved over the first secondary division mark, the setting corresponds to "310"; that is, it may represent 3.10, or .00310, or 3100. If it is moved directly over the fourth secondary division mark, the setting corresponds to "340," and this may represent 34 or .034, and so forth.

Now consider the setting "346." It will be located between the secondary division marks corresponding to "340" and "350." Inasmuch as there are five small intervals, or spaces, between the secondary marks, each small space represents two units. Hence, to locate "346," the hairline is moved three small spaces to the right of "340." Again it is emphasized that the setting for "346" may represent the numbers 34.6, 3.46, 346, .00346, and so on.

As another example, consider "313." It will be located between the secondary division marks corresponding to "310" and "320." Remembering that each small interval represents two units, "313" is set by moving the hairline one and one-half small spaces to the right of "310."

These settings are illustrated in Figure 2.3, along with other typical settings in this range.

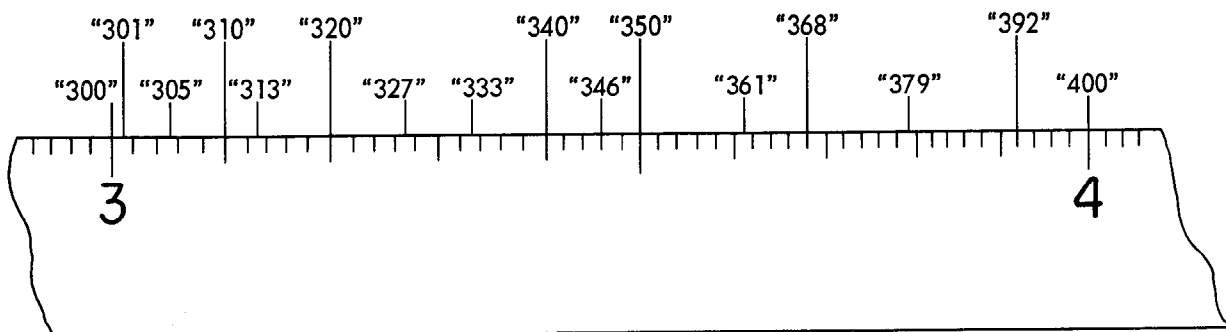


Figure 2.3

Now observe the portion of the scale between primary numbers 7 and 8, and consider the setting "733." It will be located between the secondary division marks corresponding to "730" and "740." Now if the hairline were to be moved over the small mark between "730" and "740" it would be at "735." Clearly, then, to locate "733," the hairline must be moved beyond "730" just three-fifths of the small interval between

"730" and "735." This position must be estimated, and it can be seen that it becomes difficult to account precisely for the third digit at this end of the scale.

This setting and others in this range are shown in Figure 2.4.

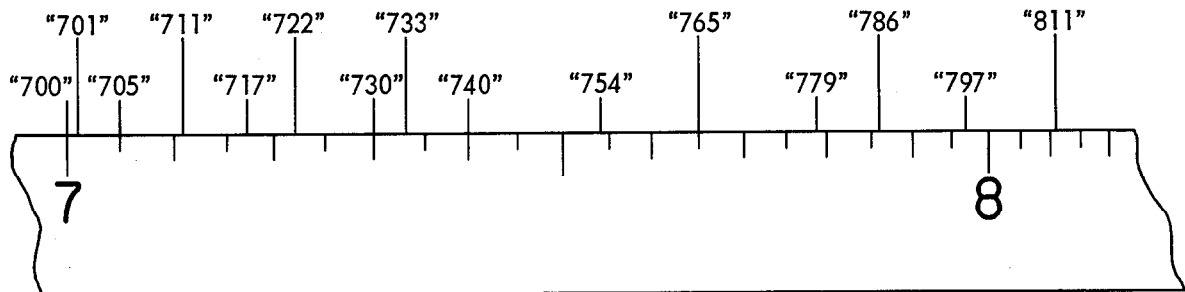


Figure 2.4

Finally, consider the left end of the scale between primary numbers 1 and 2. Because the distance between these two primary numbers is so large, the secondary division marks here are numbered with small numerals. You will note that when reading this part of the scale, it is possible to approximate a fourth digit. For example, consider the location of "1257." If the hairline is moved over the secondary division mark labeled with the small numeral 2, its position represents "1200." If it is now moved five small spaces further to the right, the reading is "1250." Finally, it must be carefully moved an additional distance estimated to be seven-tenths of the next small space. This locates "1257."

This setting and other typical settings are illustrated in Figure 2.5.

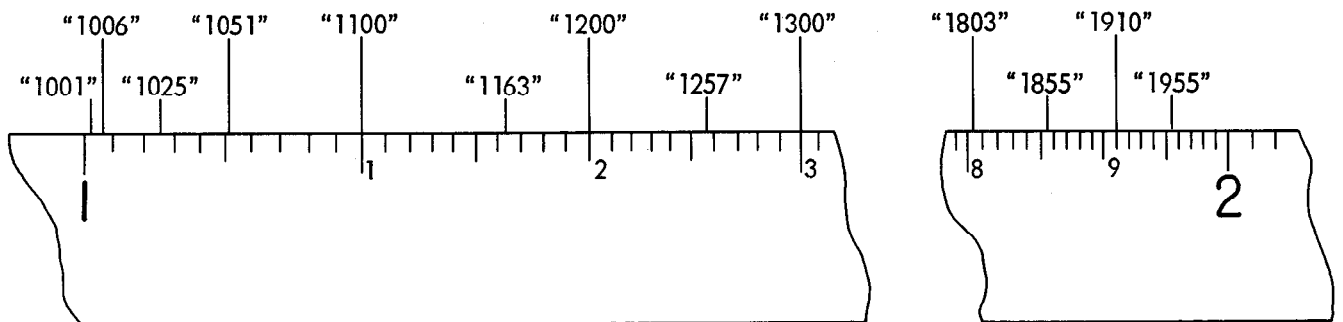


Figure 2.5

2.6 Accuracy of the slide rule

Suppose you possessed an ideal slide rule, precisely aligned and scribed, with no construction defects whatever. In reading such a rule, there would still be a possible error due to the limitation of your eye itself. On a 10-inch slide rule, the error in reading the C and D scales with the unaided eye is normally about 1 part in 1000. On a 20-inch scale, the observational error would be about 1 part in 2000, whereas on a 5-inch scale, the error would be about 1 part in 500.

Thus, on a 10-inch scale, if the observed position of the hairline is 1003, the possible error in the reading is about ± 1 , and the exact position of the hairline could be anywhere between 1002 and 1004. Again, if the observed hairline position is 313, the possible error is about ± 0.3 , and the exact position could be anywhere between 312.7 and 313.3. Finally, if the observed reading is 997, the possible error is again about ± 1 , and the exact position could be anywhere between 996 and 998.

It is apparent that for readings at the extreme left end of the scale, the fourth digit is good within ± 1 , whereas for readings at the extreme right end of the scale, the third digit is good within ± 1 . Ordinarily, the fourth digit is estimated for readings between primary marks 1 and 2; that is, numbers whose first digit is 1. All other settings are normally read to three digits only. Thus, typical slide rule readings (involving 10-inch C and D scales) would be: "1234," "602," "354," "1365," "437," "1187." To present a 10-inch scale reading as, say, "5837" would be unrealistic; here, the observational error is ± 6 , which makes the fourth digit meaningless.

Exercise 2-1

1. In Figure 2.6, read the indicated settings. Estimate the fourth digit.

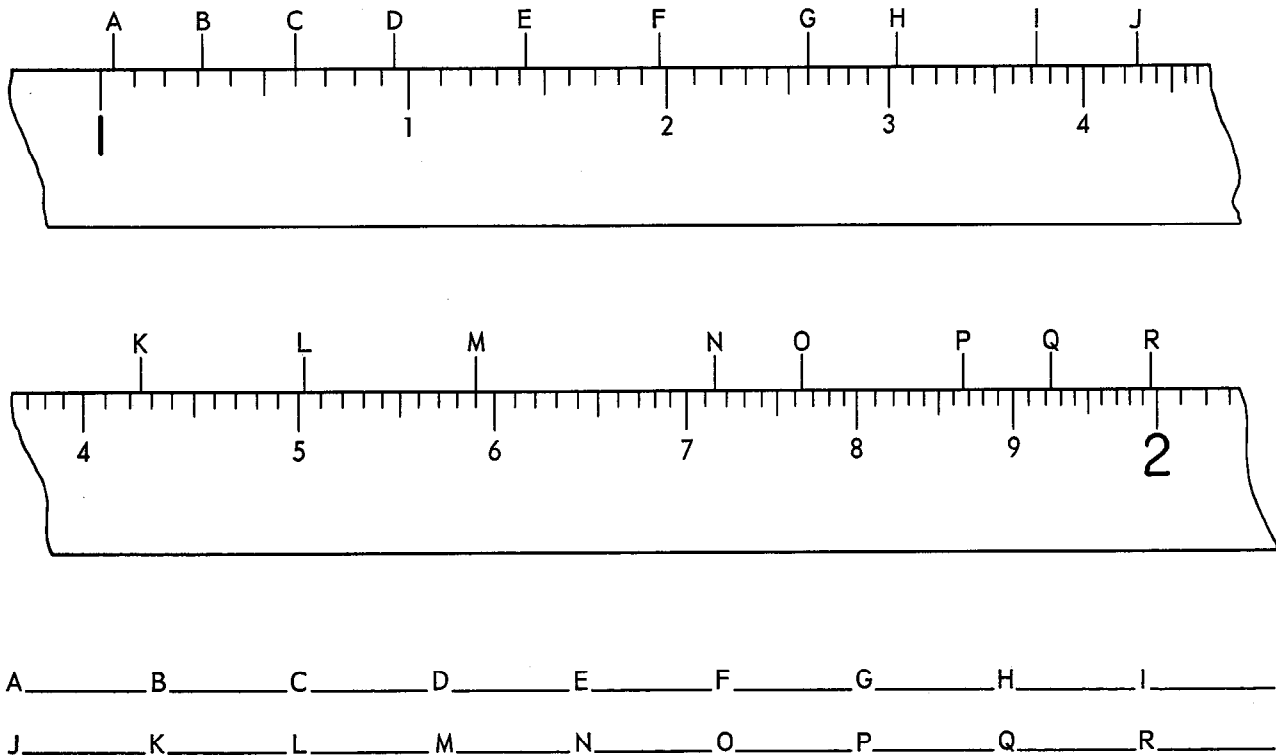


Figure 2.6

2. In Figure 2.7, read the indicated settings to three significant digits.

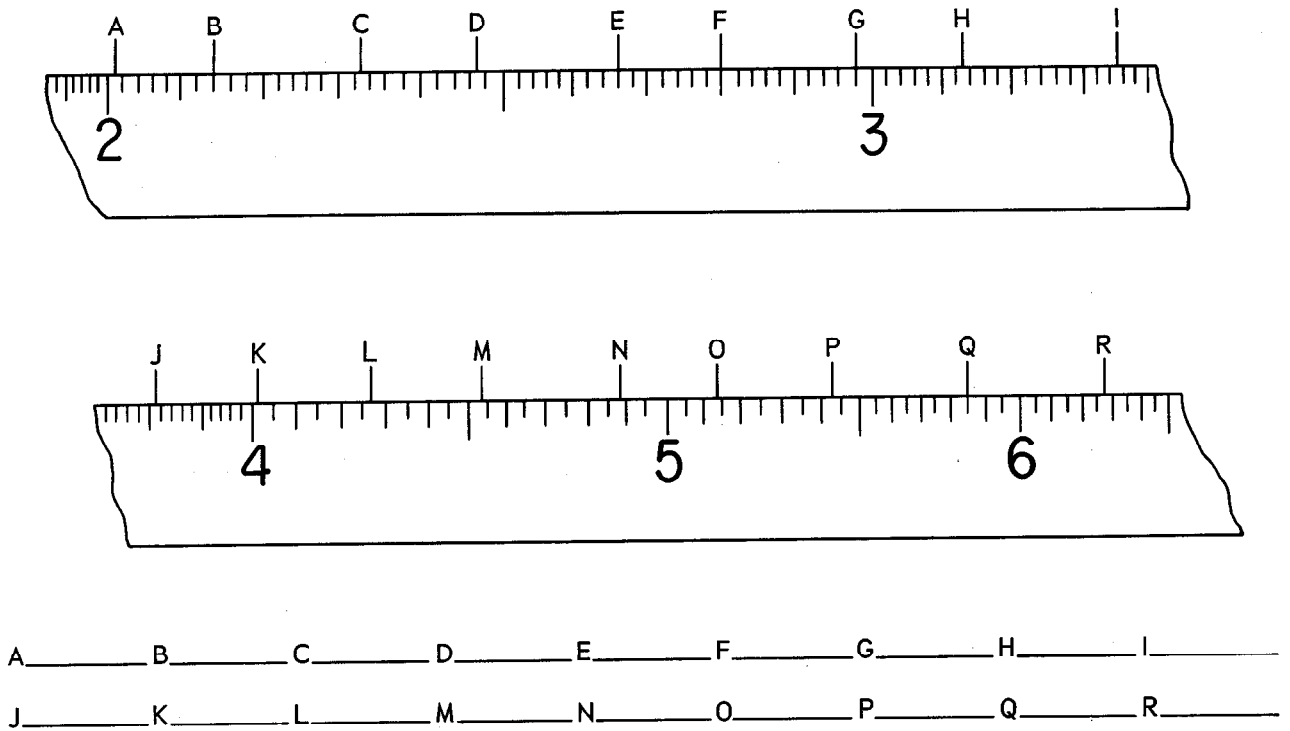


Figure 2.7

2.7 Reading the L scale

As mentioned before, the L scale is a uniform scale and, therefore, is straightforward to read. Although the principal use of this scale will be discussed in a later chapter, it may now be used to check your skill in reading the C and D scales on your own slide rule.

Examination of the L scale shows that it is divided into equal primary divisions marked 0, .1, .2, . . . , .9, 1. (On some slide rules, the decimal point is omitted before the numeral; however, it is understood to be there, whether shown or not). Each primary division is divided into ten secondary divisions each representing .01, and each secondary division is, in turn, divided into five small intervals each representing .002. A portion of the left end of the L scale, together with some typical settings, is illustrated in Figure 2.8.

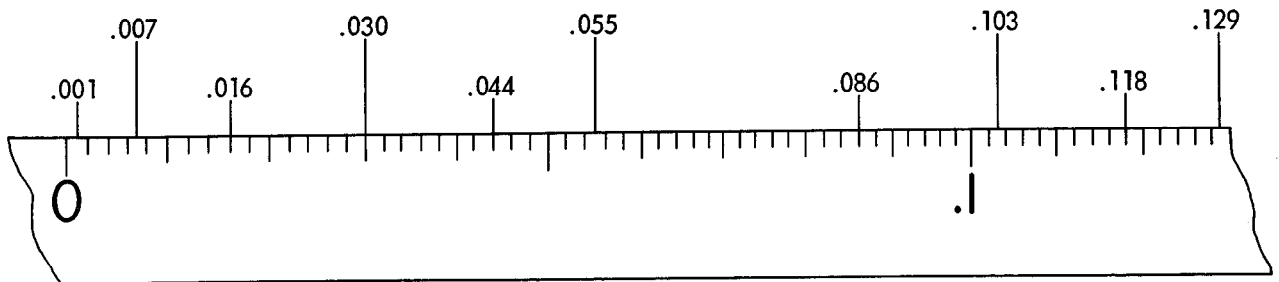


Figure 2.8

2.8 Using the L scale to check C and D readings

Let the slide rule be closed with C and D indexes exactly aligned. Now move the hairline over .312 on L, and observe the corresponding reading under the hairline on C and D. You should be reading "205" on C and D. Next, move the hairline over .617 on L; you should now be reading "414" on C and D. Finally, move the hairline over 15.45 on C and D (remember that the decimal point position does not affect the setting; the hairline is simply set at "1545"). The corresponding reading on L should now be .189.

The following exercise will further test your ability to properly read these scales.

Exercise 2-2

With rule closed (C and D indexes aligned), carefully note the corresponding readings and complete the following tables.

L	C or D
.100	"1259"
.200	
.300	
.400	
.500	
.600	
.700	
.800	
.900	
.480	
.132	
.950	
.312	
.020	
.002	
.405	

L	C or D
.989	
.266	
.810	
.006	
.850	
.030	
.517	
.998	
.389	245
	348
	107.4
	21.3
	0.398
	.0692
	15
	87,300

L	C or D
	74.4
	.00433
	100.7
	37
	10.65
	0.905
	5080
	.0101
	6.37
	4100
	.0022
	11
	96,300
	1095
	0.598
	1.765

Chapter 3

MULTIPLICATION AND DIVISION (C AND D SCALES)

3.1 Slide rule adds and subtracts lengths

The slide rule is an instrument which is designed to add or subtract lengths. If these lengths are proportional to the logarithms of numbers, it follows that adding such lengths corresponds to multiplying numbers, whereas subtracting the lengths corresponds to dividing numbers. The C and D scales are the basic scales used in these operations.

3.2 Multiplication of two numbers

The mechanics of multiplication may be illustrated by the following examples:

Example 1: $2 \times 4 = ?$ (Figure 3.1)

1. Set left index of C opposite 2 on D.
2. Move hairline over 4 on C.
3. Under hairline read "8" on D.

Essentially, we have performed the multiplication by adding logarithms:

$$\log 2 + \log 4 = \log(2 \cdot 4) = \log 8$$

More properly, we should state that the lengths which are added or subtracted are proportional to the *mantissas* of the logarithms. Thus, the slide rule combines the mantissas only and does not account for the characteristic. This is why the slide rule fails to locate the decimal point.

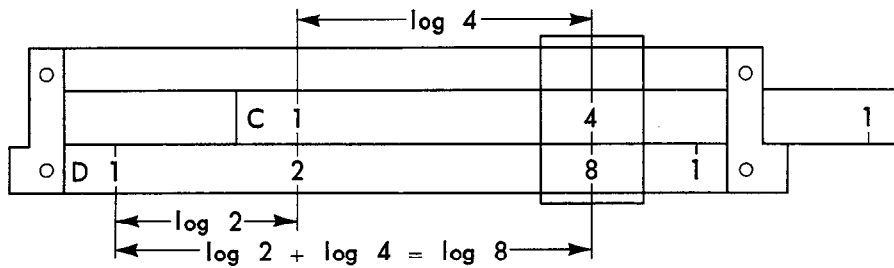


Figure 3.1

Example 2: $16 \times 3 = ?$ (Figure 3.2)

1. Set left index of C opposite 16 on D.
2. Move HL (hairline) over 3 on C.
3. Under HL read "480" on D. Answer is **48**.

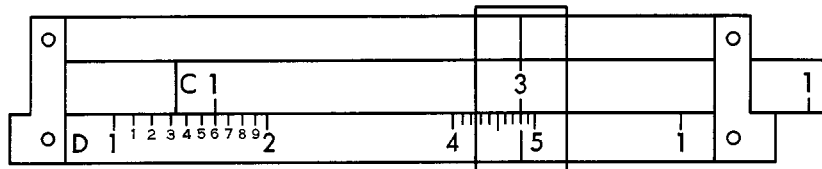


Figure 3.2

Example 3: $20.3 \times 3.86 = ?$

1. Set left index of C opposite 203 on D.
2. Move HL over 386 on C.
3. Under HL read "784" on D.

Rounding off: $20.3 \times 3.86 \approx 20 \times 4 = 80$. Answer must be **78.4**.

Example 4: $1765 \times .0000422 = ?$

1. Set left index of C opposite 1765 on D.
2. Move HL over 422 on C.
3. Under HL read "745" on D.

Shifting decimal points and rounding off:

$1765 \times .0000422 = 1.765 \times .0422 \approx 2 \times .04 = .08$. Answer must be **.0745**.

Example 5: 18.35% of 276 is ?

This corresponds to the product 0.1835×276 .

1. Set left index of C opposite 1835 on D.

2. Move HL over 276 on C.
3. Under HL read "506" on D.

Rounding off: $0.1835 \times 276 \approx 0.2 \times 300 = 60$. Answer must be **50.6**.

The foregoing examples illustrate the general procedure:

To multiply two numbers:

1. Disregard the decimal point and set the index of C over one of the numbers on D.
2. Move HL over the other number on C.
3. Under HL read answer on D.
4. Locate decimal point in answer.

Exercise 3-1

- | | |
|-----------------------------|-----------------------------------|
| 1. $1.8 \times 3.6 =$ | 11. 17.7% of 234 = |
| 2. $2.4 \times 2.2 =$ "528" | 12. $2.07 \times 3.26 =$ "675" |
| 3. $1.4 \times 5.8 =$ | 13. $10.75 \times 6.42 =$ |
| 4. $15 \times 4.7 =$ "705" | 14. 23.4% of 26.3 = "615" |
| 5. $28 \times 3.7 =$ | 15. 29.6% of 308 = |
| 6. $2.2 \times 35 =$ "770" | 16. $1.245 \times 14.7 =$ "1830" |
| 7. $1.4 \times 5.2 =$ | 17. $284 \times 1855 =$ |
| 8. 11% of 175 = "1925" | 18. $196 \times .0000207 =$ "406" |
| 9. $28 \times 3.3 =$ | 19. $1145 \times .000641 =$ |
| 10. $14 \times 2.6 =$ "364" | 20. $.000143 \times 463 =$ "662" |

3.3 Either index may be used

Consider the product 5×0.4 . If the left index of C is set opposite 5 on D, it is found that the number 4 is on the part of the C scale that projects beyond the right index of the D scale; thus it is impossible to read the answer on D. (Figure 3.3).

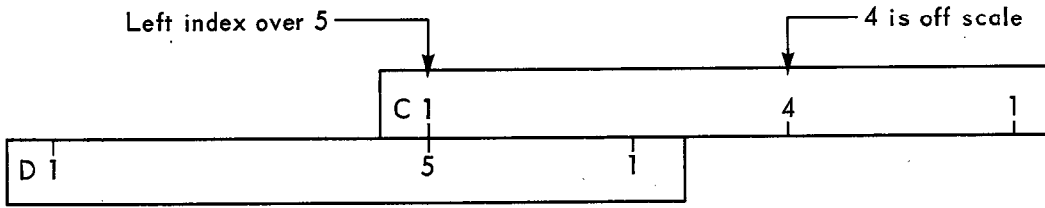


Figure 3.3

If we could somehow extend the D scale for another cycle, the answer would be located on this "extended" scale as shown in Figure 3.4.

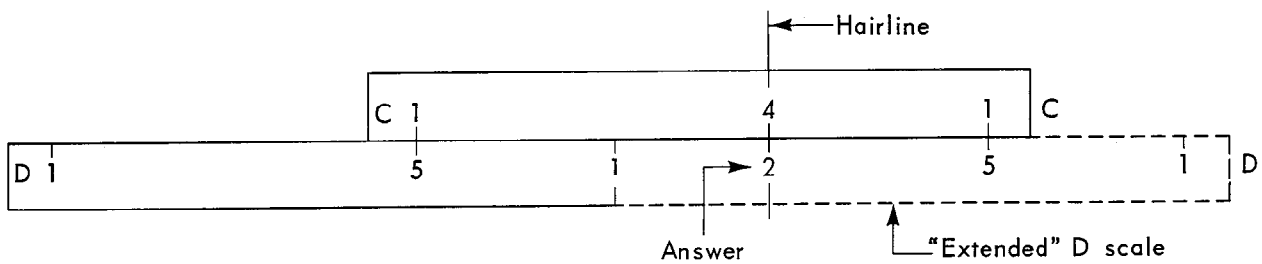


Figure 3.4

Now in Figure 3.4, note that the right index of C is opposite 5 on the "extended" scale. It is clear that the same multiplication may be accomplished by setting the *right index* of C opposite 5 on D, moving the hairline over 4 on C, and reading the result on D. This equivalent operation is illustrated in Figure 3.5.

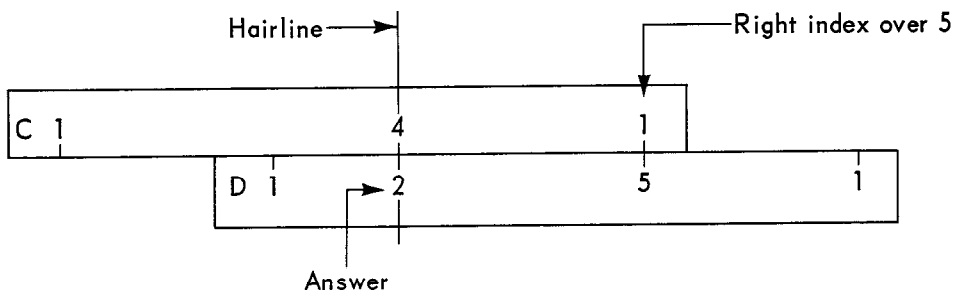


Figure 3.5

We summarize with the following statement:

When multiplying two numbers, make the first setting with *either the left or right index of C*, whichever one will ensure that the answer can be read on the D scale.

Example 1: $9 \times .04 = ?$ (Figure 3.6)

1. Set right index of C opposite 9 on D.
2. Move HL over 4 on C.
3. Under HL read "360" on D. Answer is **0.36**.

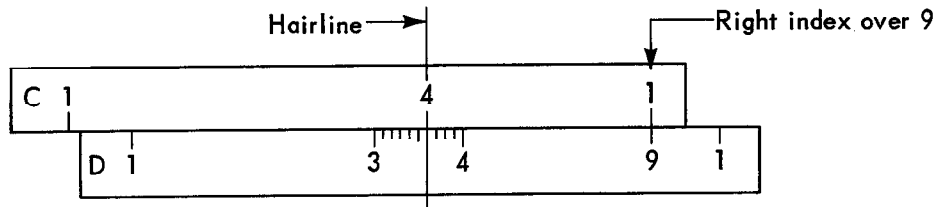


Figure 3.6

Example 2: $5.43 \times 3.26 = ?$

Again, the right index must be used.

1. Set right index of C opposite 543 on D.
2. Move HL over 326 on C.
3. Under HL read "1770" on D.

By inspection, answer must be **17.70**.

Exercise 3-2

- | | |
|------------------------------|----------------------------------|
| 1. $6.5 \times 4.2 =$ | 11. $0.74 \times 367 =$ |
| 2. $5.6 \times 5.7 =$ "319" | 12. $6.90 \times 22.4 =$ "1546" |
| 3. $8.4 \times 7.6 =$ | 13. $485 \times 2.53 =$ |
| 4. $93 \times 1.7 =$ "1581" | 14. $65.4 \times 0.392 =$ "256" |
| 5. $4.6 \times 3.2 =$ | 15. $8360 \times .0206 =$ |
| 6. 37% of 475 = "1758" | 16. $7.14 \times 64.0 =$ "457" |
| 7. 56% of 26.7 = | 17. $46.5 \times 31.2 =$ |
| 8. 43% of 3240 = "1393" | 18. $.00945 \times 50.6 =$ "478" |
| 9. $6.25 \times 3.40 =$ | 19. $.000486 \times 366 =$ |
| 10. $56 \times 40.7 =$ "228" | 20. $408 \times 71.2 =$ "290" |

3.4 Excessive extension of slide should be avoided

In multiplying two numbers, there is always a choice as to the order in which they are multiplied. Often, if the numbers are multiplied in one way the slide will extend far out of the body, whereas if the order is reversed this will not be the case. It is good practice in all slide rule operations to try to keep at least one half of the slide within the body.

Example 1: $38.4 \times .01245 = ?$

Note that if the left index of C is set opposite 384 on D, the slide extends far to the right; hence, it is better to begin by setting the index opposite 1245:

1. Set left index of C opposite 1245 on D.
2. Move HL over 384 on C.
3. Under HL read "478" on D. Answer is **0.478**.

Example 2: $2.43 \times 8.16 = ?$

Here, the right index of C must be used, and the smaller extension of the slide occurs when the right index is set opposite 8.16:

1. Set right index of C opposite 816 on D.
2. Move HL over 243 on C.
3. Under HL read "1983" on D. Answer is **19.83**.

Exercise 3-3

- | | |
|--------------------------------|---------------------------------|
| 1. $1.6 \times 23 =$ | 11. $8.2 \times 4.06 =$ |
| 2. $3.7 \times 12 =$ "444" | 12. $71.2 \times 6.44 =$ "459" |
| 3. $2.1 \times 25 =$ | 13. $21 \times 63 =$ |
| 4. $43 \times 10.2 =$ "439" | 14. $185 \times 0.74 =$ "1369" |
| 5. $5.6 \times 11 =$ | 15. $4.66 \times 14.3 =$ |
| 6. $6.3 \times 13 =$ "819" | 16. $7.24 \times 10.65 =$ "771" |
| 7. $230 \times 3.2 =$ | 17. $27.5 \times 61 =$ |
| 8. $15 \times 19 =$ "285" | 18. $4.2 \times 83.6 =$ "351" |
| 9. $27 \times 32 =$ | 19. $35.1 \times 0.22 =$ |
| 10. $5.6 \times 3.24 =$ "1814" | 20. $55 \times 3.7 =$ "204" |

21. $7.6 \times 10.45 =$
22. $43 \times 2.07 =$ "890"
23. $0.634 \times 824 =$
24. $12 \times 26.1 =$ "313"
25. $36.2 \times 0.209 =$
26. $2.57 \times 42.6 =$ "1095"
27. $3.75 \times 92.5 =$
28. $9.38 \times 18.8 =$ "1763"
29. $32.6 \times 3.15 =$
30. $25.3 \times 9.06 =$ "229"
31. $2.84 \times 56.7 =$
32. $3150 \times 2.70 =$ "851"
33. $7.33 \times 11.85 =$
34. $30.4 \times 7.56 =$ "230"
35. $0.466 \times 50.2 =$
36. $15.45 \times 8.05 =$ "1244"
37. $623 \times 0.124 =$
38. $5.77 \times 5.77 =$ "333"
39. $21.8 \times 21.8 =$
40. $2.56 \times .00304 =$ "778"
41. $.0245 \times 432 =$
42. $1015 \times .00726 =$ "737"
43. $6.32 \times .00124 =$
44. $362 \times 243 =$ "880"
45. $175 \times 63.4 =$
46. $5020 \times .00318 =$ "1596"
47. $750 \times .0222 =$
48. $1085 \times 0.986 =$ "1070"
49. $.0784 \times .001135 =$
50. $436 \times 2160 =$ "942"
51. $34,600 \times .000375 =$
52. $.00643 \times .00912 =$ "586"
53. $92.2 \times 1250 =$
54. $.000254 \times 412,000 =$ "1046"
55. $34.2 \times .00664 =$
56. $20.7 \times 87.4 =$ "1809"
57. $.00417 \times .0505 =$
58. $63,200 \times .0001095 =$ "692"
59. $.0759 \times .0759 =$
60. $(195)^2 =$ "380"
61. $.000612 \times 56.7 =$
62. 24.2% of $83.6 =$ "202"
63. 11.55% of $4310 =$
64. 75.2% of $.0905 =$ "681"
65. 43.6% of $7.62 =$
66. 2.07% of $520 =$ "1076"
67. 33.3% of $1545 =$
68. 123% of $.0842 =$ "1036"
69. 3.05% of $23,400 =$
70. 0.62% of $835 =$ "518"

3.5 Division

Division is the inverse of multiplication, and is illustrated by the following examples:

Example 1: $\frac{6}{3} = ?$ (Figure 3.7).

1. Move HL over 6 on D.
2. Slide 3 on C under HL.
3. Opposite left index of C read "200" on D. Answer is 2.

Analysing this operation, it is clear that we have simply found the number which must multiply 3 to give 6. Also from Figure 3.7, it can be seen that the answer is located at $(\log 6 - \log 3)$; hence, we have essentially performed the division by subtracting logarithms:

$$\log 6 - \log 3 = \log(6 \div 3) = \log 2$$

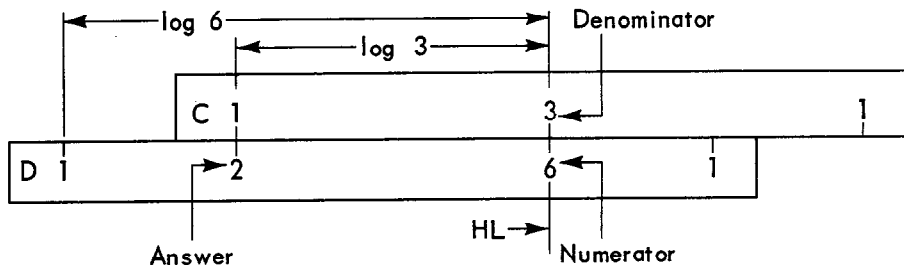


Figure 3.7

Note especially that the hairline is first moved over the *numerator* on *D*, the denominator is then pushed under the hairline on *C*, and the answer appears opposite the *C* index on *D*.

Example 2: $\frac{3}{5} = ?$ (Figure 3.8).

1. Move HL over 3 on D.
2. Slide 5 on C under HL.
3. Opposite right index of C read "600" on D. Answer is 0.6.

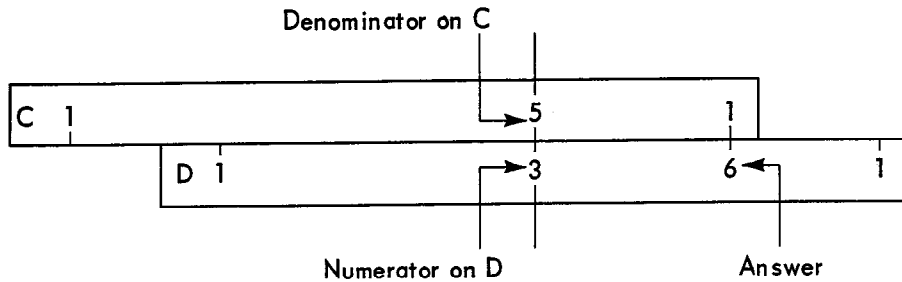


Figure 3.8

Example 3: $\frac{5.74}{3420} = ?$

1. Move HL over 574 on D.
2. Slide 342 on C under HL.
3. Opposite left index of C read "1678" on D.

Write: $\frac{5.74}{3420} = \frac{.00574}{3.42} \approx \frac{.006}{3} = .002$. Answer must be **.001678**.

Example 4: $312 \div 4.75 = ?$

1. Move HL over 312 on D.
2. Slide 475 on C under HL.
3. Opposite right index of C read "657" on D. Answer is **65.7**.

The procedure may be summarized:

To divide two numbers:

1. Disregard the decimal points, and move the hairline over the *numerator* on the *D scale*.
2. Move the slide so that the *denominator* on the *C scale* is under the hairline.
3. Read the answer on D opposite the right or left index of C, whichever is on scale.
4. Locate the decimal point in the answer.

Notice that there is only one order in which division may be performed using the C and D scales and, unlike multiplication, there is no possibility of the answer being off scale. However, there are certain combinations which will result in excessive slide extension. For example, if 11 is divided by 9, the slide must be moved almost entirely out of the body. As will be shown later, this may be avoided by using the inverted or folded scale.

Exercise 3-4

1. $\frac{62}{28} =$

3. $\frac{84}{25} =$

5. $\frac{13}{21} =$

2. $\frac{47}{19} =$ "247"

4. $\frac{364}{142} =$ "256"

6. $\frac{66}{145} =$ "455"

7. $\frac{1075}{240} =$
8. $\frac{520}{45} =$ "1155"
9. $\frac{17.6}{4.2} =$
10. $\frac{82.7}{31.6} =$ "262"
11. $\frac{143.5}{54} =$
12. $\frac{12.45}{3.26} =$ "382"
13. $\frac{282}{12} =$
14. $\frac{36}{4.1} =$ "878"
15. $\frac{756}{204} =$
16. $\frac{20.4}{5.66} =$ "360"
17. $\frac{38.4}{12.1} =$
18. $\frac{902}{23.1} =$ "390"
19. $\frac{82.4}{27.4} =$
20. $\frac{30.9}{5.24} =$ "590"
21. $\frac{1255}{63.1} =$
22. $\frac{420}{312} =$ "1346"
23. $\frac{5640}{2.03} =$
24. $\frac{67.7}{7.22} =$ "938"
25. $\frac{105.7}{82.4} =$
26. $\frac{62.7}{2.54} =$ "247"
27. $\frac{1945}{364} =$
28. $\frac{29.2}{11.43} =$ "255"
29. $\frac{71.9}{27.4} =$
30. $\frac{9.98}{5.04} =$ "1980"
31. $\frac{1}{175} =$
32. $\frac{.01}{76.6} =$ "1305"
33. $\frac{82.4}{524} =$
34. $\frac{9.46}{102} =$ "927"
35. $\frac{395}{48.2} =$
36. $\frac{26.2}{0.124} =$ "211"
37. $\frac{3.14}{16.4} =$
38. $\frac{295}{3600} =$ "819"
39. $\frac{32.4}{.0288} =$
40. $\frac{7.59}{248} =$ "306"
41. $\frac{920}{1436} =$
42. $\frac{3.24}{56,500} =$ "573"
43. $\frac{.00421}{2.17} =$
44. $\frac{754}{.00233} =$ "323"
45. $\frac{24.6}{.0824} =$
46. $\frac{368}{28.2} =$ "1305"
47. $\frac{1045}{54,200} =$
48. $\frac{.000411}{.0613} =$ "671"
49. $\frac{7240}{39.2} =$
50. $\frac{62,700}{382} =$ "1641"
51. $\frac{.00944}{543} =$
52. $\frac{237}{472,000} =$ "502"
53. $164.5 \div 3.14 =$

54. $100 \div 3750 = \text{"267"}$

55. $28.2 \div .000466 =$

56. $3.99 \div 46.2 = \text{"863"}$

57. $783 \div 9020 =$

58. $\frac{1}{49.2} = \text{"203"}$

59. $\frac{.0504}{1675} =$

60. $\frac{72,100}{.0234} = \text{"308"}$

61. $\frac{3.14}{3620} =$

62. $\frac{1000}{0.461} = \text{"217"}$

63. $\frac{27.9}{0.123} =$

64. $\frac{49,200}{32.1} = \text{"1533"}$

65. $\frac{.00723}{.0211} =$

66. $\frac{4300}{.0182} = \text{"236"}$

67. $\frac{74,600}{25.2} =$

68. $\frac{.00207}{.000523} = \text{"396"}$

69. $\frac{74.5}{4300} =$

70. $\frac{2.43}{.00071} = \text{"342"}$

71. $\frac{5640}{0.920} =$

72. $\frac{293}{.0823} = \text{"356"}$

73. $\frac{0.411}{.00624} =$

74. $\frac{9430}{.0912} = \text{"1034"}$

75. $\frac{.00593}{2.66} =$

76. 4.67 is 35% of _____

77. 34.5 is 71.4% of _____

78. 16.25 is _____% of 23.4

79. .0744 is _____% of .0913

80. 234 is 123% of _____

81. 1.345 is _____% of 12.68

82. 57.4 is _____% of 41.6

83. 6430 is 74.2% of _____

84. 307,000 is 0.214% of _____

85. .0062 is _____% of 37.6

Chapter 4

COMBINED OPERATIONS WITH C AND D SCALES

4.1 Alternating division and multiplication most efficient

Example 1: $\frac{234 \times 4.65}{312} = ?$

Here, we may first divide 234 by 312 and then multiply by 4.65:

1. Move HL over 234 on D.
2. Slide 312 on C under HL. This divides 234 by 312; the result at this point is on D under right index of C. Now to multiply by 4.65, it is only necessary to:
3. Move HL over 465 on C.
4. Under HL read "349" on D. Answer is **3.49**.

Clearly, we could have first multiplied 234 by 4.65, and then divided by 312. As will be seen, the important thing is to *alternate* the multiplication and division operations, regardless of which we choose as the initial operation.

Example 2: $\frac{16 \times 3.40}{2.5 \times 2.9} = ?$

In this case, we will first *divide* 16 by 2.5, then *multiply* this result by 3.4, and then *divide* by 2.9. The steps follow:

1. Move HL over 16 on D.
2. Slide 25 on C under HL. This divides 16 by 2.5; the result is now on D under right index of C. Next, we multiply this result by 3.4:

3. Move IIL over 34 on C. The result at this point is under the hairline on D. Finally, in order to divide by 2.9:
4. Slide 29 on C under HL.
5. Opposite right index of C, read "750" on D. Answer is **7.50**.

Note that we first move the hairline, then the slide, then the hairline, then the slide—each move accomplishing an operation. This is the most efficient way to proceed, and will always be the pattern if division and multiplication alternate with one another.

Verify the following:

$$1. \frac{143 \times 3.7}{18} = 29.4$$

$$4. \frac{8.2 \times 25}{7.1 \times 1.8} = 16.04$$

$$2. \frac{64 \times 28}{53} = 33.8$$

$$5. \frac{163 \times 6.8}{23.2 \times 4.1} = 11.65$$

$$3. \frac{260 \times 42}{147} = 74.3$$

$$6. \frac{16 \times 6.3 \times 1.7}{2.5 \times 3.7} = 18.53$$

Example 3: $\frac{12.1 \times 9.2}{8.5} = ?$

Here, if we first attempt to divide 12.1 by 8.5, the slide will extend far to the left; hence, it is better to first divide 9.2 by 8.5 and then multiply by 12.1.

Verify that the result is **13.10**.

Exercise 4-1

$$1. \frac{16 \times 4.6}{21} =$$

$$7. \frac{74 \times 19}{16} =$$

$$2. \frac{78 \times 12}{61} = \text{"1535"}$$

$$8. \frac{35.2 \times 6.24}{25.6} = \text{"858"}$$

$$3. \frac{14 \times 82}{73} =$$

$$9. \frac{2.40 \times 3.20}{1.94} =$$

$$4. \frac{210 \times 36}{125} = \text{"605"}$$

$$10. \frac{15.6 \times 6.57}{9.21} = \text{"1113"}$$

$$5. \frac{11 \times 83}{7.3} =$$

$$11. \frac{7.29 \times 500}{63.2} =$$

$$6. \frac{38 \times 4.75}{5.2} = \text{"347"}$$

$$12. \frac{2.5 \times 5.2}{3.7 \times 4.6} = \text{"764"}$$

13. $\frac{6.4 \times 25}{4.8 \times 1.5} =$
14. $\frac{15 \times 3.7}{11 \times 8.6} = \text{"587"}$
15. $\frac{6.3 \times 2.3}{1.3 \times 4.9} =$
16. $\frac{23 \times 56}{8.4 \times 19} = \text{"807"}$
17. $\frac{246 \times 52.3}{252 \times 3.6} =$
18. $\frac{2.48 \times 51.6}{4.05 \times 1.25} = \text{"253"}$
19. $\frac{2.8 \times 7.6 \times 11}{3.7 \times 5.1} =$
20. $\frac{5.6 \times 3.2 \times 9.5}{7.3 \times 4.1} = \text{"569"}$
21. $\frac{18 \times 6.2 \times 15}{13 \times 8.4} =$
22. $\frac{64 \times 17 \times 73}{39 \times 41} = \text{"496"}$
23. $\frac{8.4 \times 12 \times 6.3}{4.7 \times 1.4} =$
24. $\frac{36 \times 22 \times 4.3}{15 \times 9.5} = \text{"239"}$
25. $\frac{12.65 \times 6.44 \times 41.7}{23.6 \times 18.45} =$
26. $\frac{7.66 \times 19.25 \times 31.2}{62.1 \times 10.75} = \text{"689"}$
27. $\frac{3.6 \times 2.2 \times 5.8}{4.7 \times 1.3 \times 4.5} =$
28. $\frac{5.1 \times 9.6 \times 10.6}{7.8 \times 3.7 \times 2.3} = \text{"782"}$
29. $\frac{13 \times 4.1 \times 83}{22 \times 54 \times 1.8} =$
30. $\frac{64.7 \times 1.85 \times 0.93}{41.2 \times 5.22 \times 0.306} = \text{"1691"}$
31. $\frac{18.4 \times 7.66 \times 47.1}{25.3 \times 9.42 \times 1.76} =$
32. $\frac{.073 \times 4600}{625 \times .00375} = \text{"1432"}$
33. $\frac{5600 \times 275}{43,600 \times .0122} =$
34. $\frac{.00468 \times 6450}{3.74} = \text{"807"}$
35. $\frac{42 \times .064 \times 1.35}{310 \times .0073} =$
36. $\frac{564 \times 134 \times 0.413}{23.2 \times 174.5} = \text{"771"}$
37. $\frac{7.43 \times 4.22 \times 27.4}{82 \times 6.84 \times 4.81} =$
38. $\frac{.0263 \times 314 \times 508}{1.235 \times 861} = \text{"395"}$
39. $\frac{.0327 \times 82.1 \times 152.5}{.0662 \times 195 \times 63.1} =$
40. $\frac{.00234 \times 9640 \times 18.35}{.0582 \times 482 \times 1.036} = \text{"1425"}$
41. $\frac{294 \times 4300}{14,300} =$
42. $\frac{.00468 \times 6450}{3.74} = \text{"806"}$
43. $\frac{21.6 \times 8.75 \times 20.6}{43.5 \times 2.74 \times 5.66} =$
44. $\frac{10.75 \times 3060 \times 1250}{185 \times 216} = \text{"1029"}$
45. $\frac{.0524 \times 5400 \times 4.73}{35.2 \times 8.22 \times .0293} =$
46. $\frac{426 \times 68.2 \times 6.42}{3.14 \times 92.3} = \text{"644"}$

$$47. \frac{47.6 \times .0543 \times 17.5}{3.96 \times 28.7} =$$

$$49. \frac{.0346 \times 466 \times 22.8}{173.5 \times .0852} =$$

$$48. \frac{56.4 \times 2300 \times .0743}{37.6 \times 4.65 \times 63.1} = \text{"874"}$$

$$50. \frac{586 \times 2.67 \times 422}{72.4 \times 14.7 \times 706} = \text{"879"}$$

4.2 Alternating pattern not always possible with C and D

It often happens that you are unable to alternate the division and multiplication operations on the C and D scales. Later, you will see how the inverted and folded scales may be used to advantage in such cases; however, for the present, we will illustrate the procedure using just the C and D scales.

Example 1: $\frac{45}{2.6 \times 3.1} = ?$

1. Move HL over 45 on D.
2. Slide 26 on C under HL. We have now divided 45 by 2.6, and the result is opposite the C index on D. It now becomes necessary to move the hairline over this result on D so that we may divide it by 3.1:
3. Move HL over left index of C.
4. Slide 31 on C under HL. This divides by 3.1.
5. Opposite right index of C, read "558" on D. Answer is **5.58**.

Notice that the hairline movement in step (3) was nonoperational in the sense that it served only to mark and hold a previous result on the D scale.

Example 2: $2.4 \times 3.1 \times 4.7 = ?$

1. Set left index of C opposite 24 on D.
2. Move HL over 31 on C. We have now multiplied 2.4 by 3.1 and the result is under the hairline on D. In order to multiply again, we must first reset the C index opposite this result:
3. Slide right index of C under HL. We are now in position to multiply by 4.7:
4. Move HL over 47 on C.
5. Under HL read "350" on D. Answer is **35.0**.

Note that, in this case, the movement of the slide in step (3) was nonoperational in that it simply reset the index.

Verify the following:

$$1. \frac{75}{4.2 \times 3.6} = 4.96$$

$$2. \frac{153}{2.7 \times 5.1} = 11.11$$

3. $1.7 \times 2.8 \times 4.6 = 21.9$

5. $\frac{235}{11.5 \times 2.9 \times 3.22} = 2.19$

4. $52 \times 0.43 \times 3.6 = 80.5$

6. $\frac{3.14 \times (5.72)^2}{7.66} = 13.42$

Example 3: $\frac{27 \times 33 \times 6.2}{15 \times 19} = ?$

1. Move HL over 27 on D.
2. Slide 15 on C under HL. We have now divided 27 by 15.
3. Move HL over 33 on C. This multiplies by 33.
4. Slide 19 on C under HL. This divides by 19. We now observe that 62 on C is off-scale; hence, before multiplying by 6.2 it becomes necessary to reset the index.
5. Move HL over left index of C.
6. Slide right index of C under HL. Now multiply by 6.2:
7. Move HL over 62 on C.
8. Under HL read "1938" on D. Answer is **19.38**.

In this example, both steps (5) and (6) were nonoperational in that they served only to interchange indexes. In the following chapters you will see how these nonoperational moves may usually be eliminated by proper use of the inverted and folded scales.

Verify the following:

1. $\frac{74 \times 21 \times 43}{33 \times 17} = 119.3$

3. $\frac{2.31 \times 2.94 \times 2.62}{5.41 \times 5.17 \times 4.83} = 0.1316$

2. $\frac{17 \times 65 \times 15}{41 \times 71} = 5.69$

4. $\frac{45.1 \times 38.2 \times 184.5}{71.6 \times 67.3 \times 83.2} = 0.793$

Exercise 4-2

1. $\frac{620}{35 \times 13} =$

5. $\frac{36}{2.6 \times 7.1} =$

2. $\frac{1}{(2.3)^2} = \text{"1890"}$

6. $\frac{95}{(4.6)^2} = \text{"449"}$

3. $\frac{56}{3.4 \times 5.1} =$

7. $\frac{375}{(12.7)^2} =$

4. $\frac{135}{21 \times 4.3} = \text{"1495"}$

8. $15 \times 2.6 \times 1.9 = \text{"742"}$

9. $2.8 \times 4.9 \times 3.1 =$

10. $72 \times 0.61 \times 1.73 = \text{"760"}$

11. $3.14 \times (4.7)^2 =$

12. $3.14 \times (1.75)^2 = \text{"962"}$

13. $1.6 \times 8.2 \times 0.43 \times 2.6 =$

14. $\frac{5.2 \times 3.1 \times 6.1}{1.9 \times 1.8} = \text{"288"}$

15. $\frac{7.4 \times 26 \times 52}{3.9 \times 15} =$

16. $\frac{2.6 \times 81 \times 12}{5.3 \times 64} = \text{"745"}$

17. $\frac{33 \times 27 \times 5.6}{14 \times 17} =$

18. $\frac{195 \times 74.4 \times 13.5}{51 \times 62 \times 22} = \text{"281"}$

19. $\frac{324 \times 543 \times 224}{771 \times 635 \times 362} =$

20. $\frac{1}{2.64 \times 5.11 \times 0.344} = \text{"215"}$

21. $\frac{62.5}{3.14 \times (1.64)^2} =$

22. $\frac{340}{3.14 \times (5.5)^2} = \text{"358"}$

23. $\frac{57.2 \times 41.2}{3.14 \times (4.6)^2} =$

24. $\frac{405 \times 96}{821 \times 17.4 \times 3.83} = \text{"710"}$

25. $\frac{37.5 \times 2.61 \times 3.04}{23.1} =$

26. $3.14 \times (2.2)^2 \times 1.6 = \text{"243"}$

27. $3.14 \times (6.3)^2 \times 2.9 =$

28. $\frac{175 \times 46.3 \times 1.24 \times 0.7}{21.6} = \text{"325"}$

29. $\frac{36.4 \times 51.9}{39 \times 1.74 \times 5.7} =$

30. $\frac{100}{2.7 \times 5.1 \times 3.4} = \text{"214"}$

31. $\frac{19.2 \times 3.42 \times 2.77}{28.6} =$

32. $\frac{30.5 \times 27.6 \times 47.2}{13.15 \times 16.33 \times 1.52} = \text{"1217"}$

Chapter 5

THE INVERTED SCALE (CI)

5.1 Description of the scale

You will note that the CI scale is identical with the C scale except that it reads from right to left instead of from left to right. The C and CI scales are related in the following manner:

If the hairline is set over a number (N) on the **C** scale, the *reciprocal* of that number ($1/N$) is under the hairline on the **CI** scale.

Conversely, if the hairline is over a number on the **CI** scale, its reciprocal is under the hairline on the **C** scale.

5.2 Using the CI scale to find reciprocals

Example 1: $\frac{1}{.00412} = ?$

1. Move HL over 412 on C.
2. Under HL read "243" on CI.

Shift decimal points: $\frac{1}{.00412} = \frac{1000}{4.12}$

It is now evident that the answer must be **243**.

Example 2: Find the reciprocal of 6320.

1. Move HL over 632 on C.
2. Under HL read "1582" on CI.

$$\text{Shift decimal points: } \frac{1}{6320} = \frac{.001}{6.32}$$

Answer is **.0001582**.

5.3 The DI scale

Most modern slide rules have an inverted D scale located on the body. This scale is labeled "DI", and bears the same relation to the D scale as does the CI scale to the C scale.

Thus, if the hairline is moved over a number on the D scale, its reciprocal is under the hairline on the DI scale, and vice versa.

Exercise 5-1

Use the CI or DI scale to evaluate:

1. $\frac{1}{3.22} =$

9. $\frac{1}{723} =$

17. $\frac{1}{831} =$

2. $\frac{1}{4.75} =$

10. $\frac{1}{0.285} =$

18. $\frac{1}{.00604} =$

3. $\frac{1}{5.07} =$

11. $\frac{1}{0.413} =$

19. $\frac{1}{62.6} =$

4. $\frac{1}{2.63} =$

12. $\frac{1}{635} =$

20. $\frac{1}{283,000} =$

5. $\frac{1}{8.12} =$

13. $\frac{1}{.01725} =$

21. $\frac{1}{.0432} =$

6. $\frac{1}{9.35} =$

14. $\frac{1}{.00346} =$

22. $\frac{1}{840} =$

7. $\frac{1}{1.135} =$

15. $\frac{1}{46.2} =$

23. $\frac{1}{0.711} =$

8. $\frac{1}{32.6} =$

16. $\frac{1}{.000248} =$

24. $\frac{1}{2350} =$

5.4 Using the CI scale for division

Example 1: $234 \div 61 = ?$

This may be evaluated in the conventional manner, or it may be treated as the product: $234 \times (1/61)$. Clearly, instead of dividing by a number, we may multiply by the reciprocal of the number. Thus, the division may be accomplished as follows:

1. Set left index of C opposite 234 on D.
2. Move HL over 61 on CI. Note that HL is now over the reciprocal of 61 on C; hence, the rule is in the proper position for multiplying 234 on D by $1/61$ on C.
3. Under HL read "384" on D. Answer is **3.84**.

Example 2: $8.75 \div 1.275 = ?$

1. Set right index of C opposite 875 on D.
2. Move HL over 1275 on CI.
3. Under HL read "686" on D. Answer is **6.86**.

Verify the following divisions (use the CI scale):

- | | |
|------------------------------|------------------------------|
| 1. $1075 \div 240 = 4.48$ | 6. $920 \div 1436 = 0.641$ |
| 2. $143.5 \div 54 = 2.66$ | 7. $368 \div 28.2 = 13.04$ |
| 3. $902 \div 23.1 = 39.1$ | 8. $944 \div 543 = 1.738$ |
| 4. $5640 \div 2.03 = 2780$ | 9. $27.9 \div 0.123 = 227$ |
| 5. $105.7 \div 82.4 = 1.282$ | 10. $5640 \div 0.920 = 6130$ |

(Exercise 3-4 may be used for more drill with this method.)

5.5 Using the CI scale for multiplication

Example 1: $24.5 \times 362 = ?$

This may be treated as the quotient: $24.5 \div (1/362)$. Obviously, instead of multiplying by a number, we may divide by its reciprocal. Thus, the product may be obtained as follows:

1. Move HL over 245 on D.
2. Slide 362 on CI under HL. Notice that the reciprocal of 362 is now under the

HL on C; hence, the rule is in the proper position for dividing 24.5 on D by (1/362) on C.

3. Opposite right index of C read "887" on D.

By inspection, answer is **8870**.

Example 2: $0.327 \times 54.5 = ?$

Instead of multiplying by 54.5, we divide by the reciprocal of 54.5 as follows:

1. Move HL over 327 on D.
2. Slide 545 on CI under HL.
3. Opposite left index of C read "1782" on D.

By inspection, answer is **17.82**.

Example 3: $.00435 \times 722 = ?$

Here, again, instead of multiplying by 722, we divide by its reciprocal:

1. Move HL over 435 on D.
2. Slide 722 on CI under HL.
3. Opposite left index of C read "314" on D.

Shifting decimal points, it is seen that answer must be **3.14**.

These examples illustrate the procedure:

To multiply two numbers using CI scale:

1. Move HL over one of the numbers on **D**.
2. Move slide so that the other number on **CI** is under HL.
3. Read answer on **D** opposite **C index**.

The important feature of the CI scale is that it enables you to treat multiplication as division and vice versa. In the next chapter you will see how this scale may be used to advantage in combined operations.

Verify the following (use the CI scale):

1. $8.2 \times 4.06 = 33.3$

2. $27.5 \times 61 = 1678$

3. $43 \times 2.07 = 89.1$

7. $0.436 \times 7.62 = 3.32$

4. $32.6 \times 3.15 = 102.7$

8. $36.2 \times 0.209 = 7.56$

5. $30.4 \times 7.56 = 230$

9. $0.62 \times 835 = 517$

6. $750 \times .0222 = 16.65$

10. $.0245 \times 432 = 10.58$

(Exercise 3-3 may be used for more drill with this method.)

Chapter 6

COMBINED OPERATIONS WITH C, D, AND CI SCALES

6.1 Application of the CI scale to continued division

Proper use of the CI scale often eliminates nonoperational moves when two or more divisions occur in succession. The following examples illustrate the technique.

Example 1: $\frac{410}{2.7 \times 36} = ?$

In Chapter 4, expressions of this type were evaluated using the C and D scales only. You will recall that it was necessary to move the hairline over the result of the first division before the second division could be performed.

However, we are now in a position to eliminate this nonoperational hairline movement. First, we divide 410 by 2.7. Then, instead of dividing again by 36, we multiply by the reciprocal of 36. Thus, the sequence of operations becomes $(410) \div (2.7) \times (1/36)$. Note that this exhibits the desired alternating pattern with no nonoperational moves.

The specific steps follow:

1. Move HL over 410 on D.
2. Slide 27 on C under HL. This divides by 2.7.
3. Move HL over 36 on CI. Note that hairline is now over (1/36) on C; hence, we have effectively multiplied by (1/36). This, of course, corresponds to dividing by 36.
4. Under HL read "422" on D. Answer is **4.22**.

Verify the following:

1. $\frac{35}{19 \times 2.4} = 0.768$

2. $\frac{25}{5.4 \times 1.6} = 2.89$

$$3. \frac{630}{84 \times 35} = 0.214$$

$$5. \frac{625}{37.2 \times 41.6} = 0.404$$

$$4. \frac{16.4}{2.75 \times 3.14} = 1.899$$

$$6. \frac{100}{8.6 \times 9.2} = 1.264$$

Example 2: $\frac{152}{7.42 \times 1.85} = ?$

Here, the slide will be in better position if you first divide by 1.85, then multiply by the reciprocal of 7.42. Verify that the result is **11.07**.

Example 3: $\frac{43}{2.6 \times 6.5 \times 5.7} = ?$

The sequence of operations is: $(43) \div (2.6) \times (1/6.5) \div (5.7)$

1. Move HL over 43 on D.
2. Slide 26 on C under HL. This divides by 2.6.
3. Move HL over 65 on CI. This multiplies by $(1/6.5)$.
4. Slide 57 on C under HL. This divides by 5.7.
5. Opposite right index of C, read "446" on D. Answer is **0.446**.

Verify the following:

$$1. \frac{76.2}{12.5 \times 8.6} = 0.708$$

$$3. \frac{45}{2.7 \times 4.1 \times 1.9} = 2.14$$

$$2. \frac{164}{66 \times 2.05} = 1.211$$

$$4. \frac{2.75 \times 63.1}{4.16 \times 3.04 \times 5.27} = 2.61$$

Exercise 6-1

$$1. \frac{340}{19 \times 22} =$$

$$6. \frac{53}{3.7 \times 6.2 \times 1.8} = \text{"1284"}$$

$$2. \frac{27}{6.3 \times 2.1} = \text{"204"}$$

$$7. \frac{100}{1.6 \times 31 \times 6.8} =$$

$$3. \frac{54}{4.6 \times 7.5} =$$

$$8. \frac{370}{4.7 \times 2.8 \times 8.2} = \text{"343"}$$

$$4. \frac{19}{6.4 \times 15} = \text{"1979"}$$

$$9. \frac{64 \times 4.7}{3.8 \times 8.1 \times 1.2} =$$

$$5. \frac{27}{17 \times 4.7 \times 1.3} =$$

$$10. \frac{51}{7.2 \times 1.8 \times 7.1} = \text{"555"}$$

$$11. \frac{250 \times 23}{32 \times 1.6 \times 8.4} =$$

$$12. \frac{265}{4.7 \times 2.8 \times 8.2} = \text{"246"}$$

$$13. \frac{21.3}{36.4 \times 2.19} =$$

$$14. \frac{243}{64.2 \times 2.21} = \text{"1713"}$$

$$15. \frac{8.24}{2.17 \times 9.44} =$$

$$16. \frac{254}{18.2 \times 4.63 \times 1.35} = \text{"223"}$$

$$17. \frac{8.17}{1.94 \times 9.26} =$$

$$18. \frac{1}{5.65 \times 1.27} = \text{"1394"}$$

$$19. \frac{10}{7.82 \times 0.843} =$$

$$20. \frac{31.6 \times 68.4}{2.03 \times 9.2 \times 1.845} = \text{"627"}$$

$$21. \frac{5.02}{7.23 \times 0.175 \times 7.12} =$$

$$22. \frac{115 \times 7.65}{21.8 \times 2.74 \times 9.12} = \text{"1615"}$$

$$23. \frac{525}{7.16 \times 6.24} =$$

$$24. \frac{16.1}{270 \times .0154} = \text{"387"}$$

$$25. \frac{6350}{41.5 \times 5.06 \times 1.475} =$$

$$26. \frac{1975}{.027 \times .00564 \times 32,100} = \text{"404"}$$

$$27. \frac{1}{1.95 \times 21.6 \times 5.46} =$$

$$28. \frac{.025}{0.18 \times 2.6 \times .082 \times 3.4} = \text{"1916"}$$

$$29. \frac{10^5}{84.2 \times .0365 \times 0.475 \times 19.6} =$$

$$30. \frac{435}{29.5 \times 1.76 \times 7.64 \times 1.42} = \text{"772"}$$

$$31. \frac{803}{43 \times .0281 \times 1920} =$$

$$32. \frac{2350 \times 2.14}{.0316 \times 156.5 \times 8.4} = \text{"1210"}$$

$$33. \frac{0.254}{18.2 \times 4.83 \times 1.35} =$$

$$34. \frac{3160 \times .00584}{2.03 \times 920 \times .01845} = \text{"536"}$$

$$35. \frac{2500}{17.2 \times 26.2 \times .0462 \times 574} =$$

6.2 Application of the CI scale to continued multiplication

Example 1: $4.2 \times 5.3 \times 3.2 \times 2.7 = ?$

Here, again, expressions of this type were evaluated in Chapter 4 using only the C and D scales. You will recall that it was necessary to reset the C index after each multiplication before the next multiplication could be performed.

However, using the CI scale, we may now eliminate these extra slide movements.

In the above example, the first multiplication may be handled as a division; that is, 4.2 may be divided by the reciprocal of 5.3. We then multiply by 3.2, and finally, instead of multiplying again, we divide by the reciprocal of 2.7. The sequence of operations thus becomes: $(4.2) \div (1/5.3) \times (3.2) \div (1/2.7)$. This has the desired alternating pattern.

The steps follow:

1. Move HL over 42 on D.
2. Slide 53 on CI under HL. This divides by $(1/5.3)$ which corresponds to multiplying by 5.3.
3. Move HL over 32 on C. This multiplies by 3.2.
4. Slide 27 on CI under HL. This divides by $(1/2.7)$ which corresponds to multiplying by 2.7.
5. Opposite left index of C read "1923" on D. Answer is **192.3**.

Example 2: $\frac{462 \times .00334 \times 7.54 \times 125}{38.2} = ?$

1. Move HL over 462 on D.
2. Slide 382 on C under HL. This divides by 38.2.
3. Move HL over 334 on C. This multiplies by .00334.
4. Slide 754 on CI under HL. This multiplies by 7.54.
5. Move HL over 125 on C. This multiplies by 125.
6. Under HL read "381" on D.

Rounding off using scientific notation:

$$\frac{462 \times .00334 \times 7.54 \times 125}{38.2} \approx \frac{5 \times 3 \times 8 \times 1}{4} \times 10^{2-3+0+2-1} = 30 \times 10^0 = 30.$$

Answer is **38.1**.

Exercise 6-2

In each case, start with *division* and alternate the operations.

1. $3.2 \times 6.1 \times 2.6 =$

7. $17 \times 3.2 \times 64 =$

2. $4.5 \times 1.6 \times 1.8 = \text{"1296"}$

8. $\frac{2.1 \times 3.4 \times 4.7}{1.3} = \text{"258"}$

3. $12 \times 0.44 \times 6.3 =$

9. $\frac{4.3 \times 3.7 \times 7.2}{7.8} =$

4. $5.1 \times 4.3 \times 1.2 = \text{"263"}$

10. $\frac{5.2 \times 4.1 \times 6.6}{4.9} = \text{"287"}$

5. $2.8 \times 5.4 \times 3.2 =$

6. $3.7 \times 1.9 \times 0.85 = \text{"597"}$

11. $6.4 \times 1.8 \times 3.5 \times 1.4 =$

12. $18 \times 0.8 \times 2.3 \times 2.4 = \text{"795"}$

13. $3.7 \times 4.8 \times 1.3 \times 2.6 =$

14. $\frac{17 \times 4.4 \times 7.2 \times 2.5}{28} = \text{"481"}$

15. $\frac{5.2 \times 6.4 \times 2.7 \times 33}{85} =$

16. $\frac{35 \times 1.5 \times 3.3 \times 0.52 \times 1.3}{24} = \text{"488"}$

17. $\frac{6.1 \times 19 \times 6.5 \times 3.7}{43 \times 39} =$

18. $\frac{10.65 \times 82 \times 7.3 \times 0.24}{5.8 \times 19} = \text{"1388"}$

19. $\frac{565 \times 16.45 \times 1.23}{21.7} =$

20. $\frac{74.2 \times 6.22 \times 3.47}{5.41} = \text{"296"}$

21. $\frac{283 \times 34.6 \times .0405}{1570} =$

22. $5.42 \times 3.55 \times 1.72 \times 2.16 = \text{"715"}$

23. $(2.31)^2 \times 0.62 \times 4.87 =$

24. $3.14 \times (4.4)^2 \times 2.63 = \text{"1599"}$

25. $3.14 \times (2.13)^2 \times 37.8 =$

26. $3.14 \times (3.48)^2 \times 0.633 = \text{"241"}$

27. $23.5 \times .066 \times 383 \times .001075 =$

28. $4.52 \times 131 \times .00766 \times 4.9 \times 330 = \text{"733"}$

29. $3.22 \times 5.34 \times 12.65 \times 2.68 \times 0.845 =$

30. $210 \times 0.65 \times 314 \times .046 \times .0108 = \text{"213"}$

31. $1.5 \times 4.4 \times 0.9 \times 2.3 \times 1.4 \times 0.85 =$

32. $1.9 \times 8.5 \times 2.42 \times 0.34 \times 5.6 \times 1.4 = \text{"1042"}$

33. $\frac{0.563 \times 2.13 \times 6.15 \times 19.3}{31.5} =$

34. $\frac{175 \times 5.11 \times 0.335 \times 62.4}{234} = \text{"799"}$

35. $\frac{376 \times .0394 \times .0574 \times 14.2}{.00523} =$

36. $\frac{6.1 \times 1.9 \times (2.4)^2 \times 5.6}{1.3 \times 4.3} = \text{"669"}$

37. $\frac{54.1 \times .0322 \times 21 \times 164 \times 0.94}{890 \times 154.4} =$

38. $\frac{72.4 \times 32.5 \times 4.12 \times 0.533 \times 1.265}{51.6 \times 63.4} = \text{"1998"}$

39. $\frac{3.17 \times 7.36 \times 14.2 \times 7.08 \times 1.335 \times 0.715}{5.55 \times 2.06 \times 43.2} =$