40. $\dfrac{.0284 \times 7140 \times 0.127 \times 69 \times .00121 \times 650}{.000504 \times 23.2 \times 4.06} = \text{“294”}$

6.3 Procedure when the next setting is off-scale

In the exercises presented thus far in this chapter, it has been possible to alternate the division and multiplication operations without running off the scale. It often happens that this is not the case, and in Chapter 7 you will see how the folded scales may be used when the next setting on C or CI is off-scale.

However, the following examples illustrate how this situation may be handled using just the C, D, and CI scales.

Example 1: $\dfrac{7.4}{3.4 \times 2.1 \times 1.4} = ?$

1. Move HL over 74 on D.
2. Slide 34 on C under HL. Now observe that both 2.1 and 1.4 on CI are beyond the D scale; hence, it is impossible to multiply by the reciprocal of either one. Therefore, divide by 2.1 as follows:
3. Move HL over left index of C. This is a nonoperational move.
4. Slide 21 on C under HL. This divides by 2.1. Now multiply by the reciprocal of 1.4:
5. Move HL over 14 on CI.
6. Under HL read "740" on D. Answer is ***0.740.***

Example 2: $2.1 \times 1.8 \times 26 \times 1.7 = ?$

1. Move HL over 21 on D.
2. Slide 18 on CI under HL. Note that both 26 and 1.7 on C are beyond the D scale; hence, it is impossible to multiply by either of them. Therefore, divide by the reciprocal of 26 as follows:
3. Move HL over right index of C. This is nonoperational.
4. Slide 26 on CI under HL. Now multiply by 1.7:
5. Move HL over 17 on C.
6. Under HL read "1670" on D. Answer is ***167.0.***

It is clear from the foregoing examples that if, during a combined or continuous operation on the C-D-CI scales, a multiplication is off-scale, you may handle it as a division and proceed.

Exercise 6-3

In each of the following, start with division:

1. $\dfrac{240}{39 \times 0.78} =$

2. $\dfrac{41}{3.1 \times 12} = \text{“1102”}$

3. $\dfrac{710}{29 \times 1.6} =$

4. $\dfrac{212}{42.6 \times 5.24} =$ "950"

5. $\dfrac{63}{3.4 \times 13 \times 1.7} =$

6. $\dfrac{83}{3.9 \times 20 \times 1.6} =$ "665"

7. $\dfrac{230}{4.1 \times 6.1 \times 12} =$

8. $\dfrac{14}{2.6 \times 6.1 \times 0.76} =$ "1162"

9. $\dfrac{706}{4.32 \times 15.25 \times 11.6} =$

10. $\dfrac{83.2}{6.47 \times 1.26 \times 11.5} =$ "888"

11. $\dfrac{196}{27 \times 7.95 \times 0.844} =$

12. $5.8 \times 3.6 \times 0.71 \times 6.3 =$ "934"

13. $7.6 \times 3.5 \times 7.1 \times 0.44 =$

14. $1.9 \times 2.2 \times 1.5 \times 1.4 =$ "878"

15. $7.15 \times 3.62 \times 0.734 \times 0.845 =$

16. $\dfrac{5.3 \times 5.5 \times 6.3}{2.6} =$ "706"

17. $\dfrac{2.8 \times 1.7 \times 1.9}{6.1} =$

18. $\dfrac{7.8 \times 6.4 \times 5.7}{4.2} =$ "677"

19. $\dfrac{2.48 \times 1.34 \times 1.165}{6.07} =$

20. $\dfrac{63.4 \times 7.21 \times 0.93}{3.54} =$ "1200"

21. $\dfrac{2.8 \times 5.6}{1.3 \times 1.5} =$

22. $\dfrac{3.2 \times 18}{6.5 \times 6.7} =$ "1323"

23. $\dfrac{39.4 \times 8.66}{1.73 \times 12.4} =$

24. $\dfrac{2.15 \times 21.7 \times 1.98}{5.06 \times 6.15} =$ "297"

25. $\dfrac{8.35 \times 81.3}{4.14 \times 1.62 \times 12.6} =$

Chapter 7

THE FOLDED SCALES (CF, DF, CIF)

7.1 Description of the scales

If you examine the scales on your rule marked CF and DF, you will see that they are identical to the regular C and D scales except that they begin and end at π, thus placing the index very nearly at the center of the scale. The CIF scale is the inverse of the CF scale; that is, if the hairline is set over a number on CIF, its reciprocal will be under the hairline on CF and vice versa.

7.2 Multiplication on the folded scales

An important relationship between the regular and folded scales is the following:

If either index of **C** is set opposite a number on **D**, the index of **CF** is opposite that *same* number on **DF.**

For example, set the right index of C opposite 6 on D, and note that the CF index is also opposite 6 on DF. Suppose you now go through the following moves with your slide rule:

1. Set left index of C opposite 2 on D. Observe that CF index is now opposite 2 on DF.
2. Move HL over 4 on C. Under HL read the product, $4 \times 2 = 8$, on D.
3. Move HL over 4 on CF. Under HL read the product, $4 \times 2 = 8$, on DF.

Inasmuch as our initial setting put the C index opposite 2 on D, and at the same time put the CF index opposite 2 on DF, we were in a position to perform the multiplication on *either* set of scales, regular or folded.

Let us repeat with another illustration:

1. Set right index of C opposite 7 on D. Note that CF index is now also opposite 7 on DF.
2. Move HL over 5 on C. Under HL read the product, $7 \times 5 = 35$, on D.
3. Move HL over 5 on CF. Under HL read 35 on DF.

7.3 Multiplying a number successively

Example 1: Given the equation $y = (2.3)x$. Find the corresponding values of y when x takes the values: 1.6, 3.8, 5.4, and 8.5.

1. Set left index of C opposite 23 on D. CF index is now also opposite 23 on DF.
2. Move HL over 16 on C. Under HL read "368" on D.
3. Move HL over 38 on C. Under HL read "874" on D.
   Observe that 54 is off-scale on C; hence, we now go to the folded scales:
4. Move HL over 54 on CF. Under HL read "1242" on DF.
5. Move HL over 85 on CF. Under HL read "1955" on DF.

Results are: ***3.68***, ***8.74***, ***12.42***, and ***19.55***.

Example 2: Given the relation, $E = 2.47R$. Calculate E when R takes the successive values: 12.5, 27.6, 46.3, 78.2, and 145.

1. Set left index of C opposite 247 on D.
2. Move HL over 125 on C. Under HL read "309" on D.
3. Move HL over 276 on C. Under HL read "682" on D.
4. Note that 463 is off-scale on C; hence, move HL over 463 on CF. Under HL read "1144" on DF.
5. Move HL over 782 on CF. Under HL read "1931" on DF.
6. Move HL over 145 on C. Under HL read "358" on D.

The corresponding values of E are: ***30.9***, ***68.2***, ***114.4***, ***193.1***, and ***358***.

Exercise 7-1

1. Given the formula, $R = 3.06T$. Find R corresponding to the following values of T: 1.84, 2.77, 5.75, and 8.24.
2. Given the formula, $P = .0534Q$. Find P corresponding to the following values of Q: 21.4, 56.2, 111, 175, and 321.
3. Given the equation, $y = (1.34)x$. Find the corresponding values of y when x takes the values: 1.25, 2.75, 3.50, 4.75, and 7.25.
4. Given the equation, $y = (0.635)x$. Find corresponding values of y when x takes the values: 1.25, 2.75, 5.45, 14.75, and 22.6.

5. A certain distance-time relationship is known to be $D = (3.06)T$, where D is distance in meters and T is time in seconds. Find D corresponding to the following values of T: 1.24, 2.35, 3.10, 4.66, 7.25, and 10.45.
6. Find 36.4% of the following numbers: 12.5, 39.0, 22.4, 185, 211, and 766.
7. Given the relationship, $R = (0.269)L$, where L represents the length of a wire in feet, and R represents the resistance in ohms. Find R corresponding to the following lengths: 150′, 320′, 535′, 820′, 1150′, 2040′, and 4050′.
8. A certain gasoline weighs 5.63 lbs per gallon. Find the weights of the following amounts: 12.5 gals, 28.4 gals, 56.7 gals, 82.4 gals, 130 gals, and 211 gals.
9. A firm is allowed a 45% discount off list on a certain item. Find the firm's net prices corresponding to the following list prices: $4.56, $8.20, $34.20, $11.50, $204.00, and $107.00. (Slide rule accuracy only.)
10. An alloy is 23.2% copper by weight. Find the pounds of copper in each of the following amounts of alloy: 140 lbs, 275 lbs, 560 lbs, 725 lbs, 1120 lbs, and 3060 lbs.
11. Given the relationship $e = (.000148)T$, where e is the stretch in inches of a steel wire, and T is the pull on the wire in lbs. Find e corresponding to the following values of T: 2500 lbs, 4250 lbs, 6100 lbs, 7800 lbs, 11,500 lbs, and 26,000 lbs.
12. To determine selling price, a merchant marks up his merchandise $37\frac{1}{2}$% based on cost. Find the selling prices corresponding to the following cost prices: $0.36, $1.74, $0.82, $5.42, $.094, $0.26, and $3.24. Give selling price to nearest cent.
13. If a firm gets a trade discount of 20% less 15% less 15%, then the following formula applies: $N = (.80)(.85)(.85)L$, where L is the list price, and N is the net price. Find N corresponding to the following values of L: $10.50, $150.00, $43.50, $7.45, $16.50, and $340.00. (Slide rule accuracy only).
14. Given the formula: $V = \dfrac{\pi(2.83)N}{30}$, where V is rim speed in ft per sec., and N is angular speed in rpm. Find V corresponding to the following values of N: 750, 1830, 2400, 5500, and 12,500.

7.4 Dividing a number successively

Example: Given the relation, $P = (56.5)/V$. Calculate P when V takes the values: 17.2, 38.5, 61.4, 81.0, and 106.

When we wish to divide successively by several numbers, it is more convenient to multiply by the reciprocals.

1. Set right index of C opposite 565 on D.
2. Move HL over 172 on CI. Under HL read "328" on D.
3. Move HL over 385 on CI. Under HL read "1468" on D.
4. Observe that 614 is off-scale on CI; hence, move HL over 614 on CIF. Under HL read "920" on DF.
5. Move HL over 810 on CIF. Under HL read "698" on DF.
6. Move HL over 106 on CIF (or CI). Under HL read "533" on DF (or D).

Values of P are: ***3.28, 1.468, 0.920, 0.698***, and ***0.533***.

Exercise 7-2

1. Given the equation, $y = 16/x$. Find y when x takes the values: 2.5, 4.5, 7.5, 10.5, and 14.5.
2. Given the equation, $y = 75/x$. Find y when x takes the values: 2.2, 3.3, 6.6, 7.7, and 8.8.
3. Given the equation, $y = (2450)/x$. Find y when x takes the values: 12.5, 16.7, 24.6, 43.2, and 95.0.
4. Given the equation, $y = (520)/x$. Find y when x takes the values: 15, 25, 34, 58.5, 74.5, 95, 145, and 225.
5. Given the relationship, $I = (12.5)/R$. Find I when R takes the values: 2.5, 35, 150, 435, and 1650.
6. Given the rate-time formula, $r = (47.5)/t$. Find r when t takes the values: 0.75, 2.65, 4.35, 13.6, 54.5.
7. The total investment necessary to return $2250 annually is given by the formula, $P = (2250)/r$, where r is the annual interest rate. Find P when r takes the values: .025, .045, .075, .125, and .150.
8. Given the formula, $N = (5400)/R$, where N is the rpm of a belt-driven pulley, and R is the radius of the pulley in inches. Find N corresponding to the following values of R: 3.25″, 4.25″, 6.50″, 8.75″, and 10.5″.
9. Given the temperature-time relationship, $T = \dfrac{360}{t} + 20$, where T is temperature in degrees Centigrade, and t is time in minutes. Find T corresponding to the following values of t: 25, 55, 125, 325, and 1500.
10. Given the formula, $B = \pi(2.7)^2/W$. Find B corresponding to the following values of W: 1.055, 1.645, 2.06, 4.85, and 7.22.

7.5 Other operations involving the folded scales

The preceding exercises have shown that if, during an operation on the regular scales, a hairline setting is off-scale, the setting may normally be made on the folded scales. Conversely, if the operation is proceeding on the folded scales and a hairline position is beyond the scale, the setting may usually be found on the regular scale. Therefore, proper use of the folded scales will eliminate the nonoperational moves which are often required when operating with just the C, D, and CI scales.

Example 1: $\dfrac{28 \times 13}{41} = ?$

1. Move HL over 28 on D.
2. Slide 41 on C under HL. The result of this division is now opposite the right index of C on D, and is also opposite the CF index on DF. Hence, the next multiplication may be carried out on either the regular or the folded scales.
3. Observe that 13 is off-scale on C; therefore, move HL over 13 on CF.
4. Under HL read "888" on DF. Answer is ***8.88***.

Example 2: $\dfrac{45 \times 5.70}{1.8 \times 1.2} = ?$

1. Move HL over 45 on D.
2. Slide 18 on C under HL. Now, as before, the next multiplication may be carried out on either the regular or folded scales. We note that 57 is off-scale on C; hence, we go to the folded scales:
3. Move HL over 57 on CF. Result at this point is under HL on DF; therefore, the next division must be made using the CF scale:
4. Slide 12 on CF under HL. Answer is now opposite CF index on DF, and is also opposite C index on D. This will always be the case; that is, whenever the result is opposite an index, it may be read on either DF or D. You will probably prefer reading on D.
5. Opposite left index of C read "1188" on D. Answer is ***118.8.***

Verify the following:

1. $\dfrac{29 \times 6.5}{14} = 13.47$
2. $\dfrac{23 \times 143}{42} = 78.3$
3. $\dfrac{48 \times 1.7}{8.3} = 9.83$
4. $\dfrac{35.2 \times 22.6}{8.41 \times 12.5} = 7.57$
5. $\dfrac{173 \times 154}{276 \times 77} = 1.254$
6. $\dfrac{26.4 \times 11.4}{4.18 \times 9.25} = 7.79$

Example 3: $\dfrac{48}{2.1 \times 1.1} = ?$

1. Move HL over 48 on D.
2. Slide 21 on C under HL. Now we must multiply by the reciprocal of 1.1. Observe that 11 is off-scale on CI; hence, go to the folded scales:
3. Move HL over 11 on CIF. Under HL read "208" on DF. Answer is ***20.8.***

Verify the following:

1. $\dfrac{22}{3.8 \times 6.4} = 0.905$
2. $\dfrac{175}{4.1 \times 5.6} = 7.62$
3. $\dfrac{68}{2.7 \times 1.55} = 16.25$
4. $\dfrac{280}{16.5 \times 12.3} = 1.380$
5. $\dfrac{56.3}{2.77 \times 1.085} = 18.74$
6. $\dfrac{183}{3.62 \times 5.74} = 8.81$

Example 4: $\dfrac{5200 \times 630}{2.9 \times .023 \times 0.27} = ?$

1. Move HL over 52 on D.
2. Slide 29 on C under HL.
3. Note that 63 is off-scale on C; hence, move HL over 63 on CF. The next division must be made on the folded scales:
4. Slide 23 on CF under HL. The result at this point is opposite CF index on DF, and is also opposite C index on D; therefore, the next multiplication may be carried out on either the folded or regular scales. In this case we must multiply by the reciprocal of 2.7.
5. Observe that 27 on CIF is off-scale; hence, move HL over 27 on CI.
6. Under HL read "1819" on D.

Rounding off using scientific notation:

$$\frac{\overset{(3)}{5200} \times \overset{(2)}{630}}{\underset{(0)}{2.9} \times \underset{(-2)}{.023} \times \underset{(-1)}{0.27}} \approx \frac{5 \times 6}{3 \times 2 \times 3} \times 10^{3+2-0+2+1} = \frac{5}{3} \times 10^{8} \approx 1.7 \times 10^{8}$$

Answer must be $\boldsymbol{1.819 \times 10^{8}}$.

It should now be clear that, after an operation which involves moving the slide, the hairline may then be positioned on either the regular or folded scales. If the regular scale is chosen, the operation must continue on the regular scales until the next hairline movement when, again, a choice may be made. If the folded scale is chosen, the operation must continue on the folded scales until the next hairline movement when, again, a choice exists.

It is important, then, to keep in mind the following:

> During a combined or continued operation, one may shift from the regular to the folded scales, or vice versa, *only when the hairline is moved.*

Verify the following:

1. $\dfrac{55 \times 13}{8.6 \times 6.8 \times 3.5} = 3.49$
2. $\dfrac{75 \times 53 \times 29}{36 \times 47} = 68.1$
3. $\dfrac{56}{31 \times 17 \times 1300} = 8.18 \times 10^{-5}$
4. $\dfrac{2.32 \times 6.25 \times 1.15}{4.71 \times 3.92} = 0.903$
5. $\dfrac{615}{3.14 \times 1.45 \times 1.45} = 93.1$
6. $\dfrac{136.5 \times 726}{0.273 \times .0514 \times 0.84} = 8.41 \times 10^{6}$

Example 5: $3.3 \times 4.1 \times 0.84 \times 0.76 \times 5.5. = ?$

1. Move HL over 33 on D.
2. Slide 41 on CI under HL.
3. Move HL over 84 on CF.
4. Slide 76 on CIF under HL. Note that 55 is now on-scale for both regular and folded scales. Suppose we choose the regular:
5. Move HL over 55 on C.
6. Under HL read "475" on D. Answer is ***47.5.***

Verify the following:

1. $1.8 \times 2.8 \times 1.4 \times 2.3 = 16.23$
2. $\dfrac{55.5 \times 15.4 \times 7.45 \times 5.2}{263} = 126.0$
3. $3.2 \times 2.1 \times 1.3 \times 1.2 \times 4.7 = 49.3$
4. $\dfrac{3.14 \times 2.06 \times (6.25)^2}{7.62} = 33.2$

Example 6: $\dfrac{11}{9.1 \times 0.85} = ?$

Here, the slide will be in better position if the operation starts on the folded scales. Verify that the result is ***1.422.***

Verify the following:

Start each operation on the folded scales:

1. $\dfrac{110}{8.2 \times 4.1} = 3.27$
2. $\dfrac{8.6}{1.3 \times 1.7} = 3.89$
3. $\dfrac{112 \times 14}{83} = 18.90$
4. $\dfrac{13.6 \times 12.4}{9.2 \times 5.6} = 3.27$
5. $\dfrac{94}{12.3 \times 3.72 \times 2.8} = 0.733$
6. $\dfrac{115 \times 1.08 \times 2.16}{9.15} = 29.3$

The techniques which have been illustrated thus far are basic, and they must be mastered if you are to use your slide rule with greatest efficiency. Many users of the slide rule have never developed the habit of properly utilizing the reciprocal and folded scales. This means that they make more moves and settings than necessary, which, in turn, increases the chance for error. It is important that you force yourself to use the reciprocal and folded scales until it becomes second nature; you should be just as comfortable and confident with these scales as with the C and D scales.

The following exercise set is designed to give you more practice with combined operations. See if you can go through them without making any nonoperational moves.

Exercise 7-3

1. $\frac{5.6 \times 7.8}{2.5} =$

2. $\frac{22}{4.7 \times 7.1} =$ "659"

3. $4.1 \times 4.9 \times 0.77 =$

4. $\frac{19 \times 21}{48 \times 13} =$ "640"

5. $\frac{12 \times 6.4 \times 1.7}{9.3} =$

6. $\frac{33}{(1.7)^2 \times 6.4} =$ "1785"

7. $1.9 \times 2.4 \times 2.1 \times 0.74 =$

8. $\frac{8.7 \times 21 \times 5.3}{11 \times 7.6} =$ "1158"

9. $\frac{37 \times 26}{81 \times 7.2} =$

10. $\frac{9.3 \times 8.4}{1.1 \times 1.3 \times 2.7} =$ "202"

11. $\frac{12 \times 10.5 \times 2.9}{8.8 \times 9.4} =$

12. $\frac{7.8 \times 2.1 \times 14}{3.5 \times 9.3} =$ "705"

13. $\frac{2.5 \times 4.9 \times 3.1}{1.6 \times 2.3} =$

14. $\frac{4.2 \times 27 \times 3.2}{6.3 \times 5.9} =$ "976"

15. $\frac{42 \times 4.5 \times 0.52}{17} =$

16. $\frac{76}{10.5 \times 1.2 \times 1.5} =$ "402"

17. $\frac{130}{8.6 \times (0.92)^2} =$

18. $4.1 \times (5.2)^2 =$ "1109"

19. $\frac{28 \times 19}{58 \times (2.1)^2} =$

20. $(5.2)^3 \times 1.25 =$ "1757"

21. $\frac{29 \times 32}{9.75 \times 9.3} =$

22. $\frac{74.1 \times 6.55 \times 9.25}{4.26} =$ "1054"

23. $\frac{19.6 \times 1.12 \times 2.43}{7.95 \times 0.844} =$

24. $\frac{83.2}{1.26 \times 6.47 \times 11.5} =$ "888"

25. $\frac{17.2 \times 120}{28.4 \times 34.2 \times 6.05} =$

26. $\frac{78.2}{(1.12)^3} =$ "557"

27. $\frac{46.5}{(2.1)^3} =$

28. $\frac{100}{4.52 \times 3.24 \times 1.85} =$ "369"

29. $\frac{62.5 \times 5.94 \times 6.47}{10.3 \times 1.13 \times 1.245} =$

30. $\frac{12.6 \times 15.7}{4.82 \times 7.11 \times 0.627} =$ "920"

31. $\frac{12.65 \times 6.21 \times 2.77}{8.34 \times 4.56 \times 3.69} =$

32. $\frac{24.6 \times 152}{3.85 \times 4.27 \times 6.23 \times 5.76} =$ "634"

33. $\frac{51.7 \times 64.7}{20.6 \times 2.35 \times 5.68 \times 17.2} =$

34. $7.12 \times 0.645 \times 4.73 \times 5.25 =$ "1140"

35. $5.84 \times 3.6 \times 0.211 \times (1.43)^2 =$

36. $\dfrac{47.8 \times 96.4 \times 12.4}{24.3 \times 5.62 \times 7.07 \times 19.2} =$ "308"

37. $\dfrac{.0364 \times 16.45}{723} =$

38. $\dfrac{485}{18.3 \times 2160} =$ "1227"

39. $.00264 \times .0663 \times 0.824 =$

40. $\dfrac{2650 \times .0624}{138 \times 149} =$ "804"

41. $\dfrac{926 \times 82.6}{.0113 \times 132 \times 264} =$

42. $\dfrac{.00473 \times 1150}{5.72 \times .0862} =$ "1103"

43. $\dfrac{.0224 \times 486 \times .00308}{0.154 \times .0196} =$

44. $\dfrac{.0226 \times 1940 \times 17.3}{845} =$ "898"

45. $\dfrac{184 \times 11.6 \times 23.6}{7840 \times 836} =$

46. $.00314 \times 2960 \times 52.6 \times .0345 =$ "1687"

47. $\dfrac{1}{13.6 \times (.0264)^2} =$

48. $\dfrac{718 \times .000617 \times 75.2}{.0246} =$ "1355"

49. $392 \times (.0616)^2 =$

50. $\dfrac{28.2}{.00416 \times 7.22 \times 32.4} =$ "290"

51. $\dfrac{.00743 \times (46.2)^2}{21.7} =$

52. $59.2 \times .00134 \times 410 \times 0.512 \times 8.3 =$ "1382"

53. $\dfrac{4.01 \times 232 \times 1.345}{96.8 \times 16.75} =$

54. $\dfrac{1}{.0764 \times 123 \times 2.75} =$ "387"

55. $\dfrac{16{,}000}{46.2 \times 9.57 \times 100.4} =$

56. $\dfrac{.0743 \times 575}{.00524 \times 8540 \times 17.5 \times 10^{-3}} =$ "546"

57. $\dfrac{.0816 \times 1075 \times 5.20 \times .0625}{426 \times .00347} =$

58. $\dfrac{6420 \times 10^{4}}{1735 \times 24.6 \times 402} =$ "374"

59. $752 \times 14.65 \times 247 \times 60.3 =$

60. $\dfrac{4.62 \times 1575 \times .00288}{362 \times 5.71 \times 23.7 \times 10^{-3}} =$ "428"

7.6 Multiplication and division by π

Inasmuch as the folded scales begin and end at π, the following relationship holds:

> If the hairline is moved over a number N on the **D** scale, then the product ($\pi \times N$) will be under the hairline on **DF.**
>
> Conversely, if the hairline is moved over a number N on **DF,** the quotient ($N \div \pi$) will be under the hairline on **D.** The C and CF scales are similarly related.

This means that, given the diameter, we can find the circumference of a circle with a single setting. Also, we may eliminate one operation when evaluating combined expressions involving π.

Example 1: Given circles with diameters 2.43 inches, 5.75 inches, and 12.6 inches. Find the respective circumferences.

1. Move HL over 243 on D. Under HL read "764" on DF.
2. Move HL over 575 on D. Under HL read "1806" on DF.
3. Move HL over 126 on D. Under HL read "396" on DF.

Answers are: ***7.64*** in., ***18.06*** in., and ***39.6*** in.

Example 2: $\frac{\pi \times 6.84}{8.31} = ?$

1. Move HL over 684 on D. Now $\pi \times 6.84$ is under HL on DF; hence, we need only divide by 8.31 on the folded scales:
2. Slide 831 on CF under HL.
3. Opposite left index of C, read "258" on D. Answer is ***2.58.***

Example 3: $\frac{13}{\pi \times 0.73} = ?$

1. Move HL over 13 on DF. Note that $13 \div \pi$ is now under HL on D; hence we continue on the regular scales:
2. Slide 73 on C under HL.
3. Opposite right index of C, read "567" on D. Answer is ***5.67.***

Exercise 7-4

1. Find circumferences of circles with following diameters: 2.68, 1.77, 7.05, 0.421, .0543, 17.6, 50.2, and 1.155.
2. Find diameters of circles with following circumferences: 20.4, 14.2, 83.7, 178.5, 0.417, 2430, .0206, and 1.09.
3. $\frac{36.1 \times \pi}{4.75} =$
4. $\frac{\pi \times 26}{7.2} =$ "1134"
5. $\frac{4.5 \times \pi}{23} =$
6. $\frac{\pi \times 1.73}{2.44} =$ "223"
7. $\pi \times 6.4 \times 1.3 =$
8. $\pi \times 27.3 \times 0.44 =$ "377"
9. $\pi \times (1.9)^2 =$
10. $\pi \times (4.1)^2 =$ "528"
11. $\frac{123}{\pi \times 6.34} =$
12. $\frac{17.2}{\pi \times 2.4} =$ "228"

13. $\dfrac{275}{\pi \times 8.46} =$

14. $\dfrac{56.2}{\pi \times 2.34} =$ "765"

15. $\dfrac{92.4}{\pi \times (4.75)^2} =$

7.7 Formula types

When substituting into formulas, we must often evaluate expressions similar to the following:

Example 1: $$\frac{26.5}{5.40 - (6.25 \times 0.123)} = ?$$

1. Verify that $6.25 \times 0.123 = 0.769$.
2. Expression now becomes:

$$\frac{26.5}{5.40 - 0.769} = \frac{26.5}{4.631} \approx \frac{26.5}{4.63}$$

Note that for slide rule computation, we round off the denominator to three significant figures.

3. Verify that answer is ***5.72.***

Example 2: $$\frac{244}{4.60\left[2.30 + \dfrac{6.70}{8.20}\right]} = ?$$

1. Verify that $\dfrac{6.70}{8.20} = 0.817$
2. Expression now becomes:

$$\frac{244}{4.60(2.30 + 0.817)} = \frac{244}{4.60 \times 3.117} \approx \frac{244}{4.60 \times 3.12}$$

3. Verify that answer is ***17.00.***

Example 3: $$\frac{22.2}{\dfrac{1}{2.6} + \dfrac{1}{3.2} + \dfrac{1}{4.8}} = ?$$

1. Use reciprocal scale to verify that:

$$\frac{1}{2.6} = 0.385; \frac{1}{3.2} = 0.312; \frac{1}{4.8} = 0.208$$

2. Expression becomes:

$$\frac{22.2}{0.385 + 0.312 + 0.208} = \frac{22.2}{0.905}$$

3. Verify that answer is ***24.5.***

Exercise 7-5

1. $\frac{41}{1 + (3.2 \times 1.7)} =$

2. $\frac{120}{1 - (45 \times .016)} =$

3. $7.60 \left[12.40 + \frac{36.3}{7.22}\right] =$

4. $\frac{520}{6.70 + \frac{3.25}{1.66}} =$

5. $\frac{28.6}{2.77 + \frac{1500}{3.62}} =$

6. $\frac{375}{2.88 \left[1.84 + \frac{32.6}{45.2}\right]} =$

7. $\frac{47{,}500}{165 \left[.027 + \frac{1}{230}\right]} =$

8. $\frac{1}{.00224 \left[2.63 - \frac{375}{162}\right]} =$

9. $\frac{1}{\frac{1}{18} + \frac{1}{35}} =$

10. $\frac{67.3}{\frac{1}{3.72} + \frac{1}{4.23} + \frac{1}{7.25}} =$

11. $\dfrac{.0865}{\dfrac{1}{450}+\dfrac{1}{175}+\dfrac{1}{620}} =$

12. $\dfrac{164}{15.2}\,[17.2 + (136 \times 0.245)] =$

13. $\dfrac{254}{6450}\,[368 - (26.6 \times 2.77)] =$

14. $(5.22 - 2.68)\left[37.2 + \dfrac{136}{(5.22)^2}\right] =$

15. $(2.75 - 1.37)\left[165 + \dfrac{237}{(2.75)^2}\right] =$

16. $26.5\left[\dfrac{3.72 \times 3.16}{3.72 - 3.16}\right] =$

17. $3.27 \times 10^{-6}\left[\dfrac{.01465 \times .01452}{.01465 - .01452}\right] =$

18. $\dfrac{16.3}{(1.72)^2}\left[\dfrac{3.75}{2.24} - \dfrac{3.25}{2.76}\right] =$

19. $\dfrac{2}{(.073)^2}\left[\dfrac{1.68}{75.2} - \dfrac{1.44}{86.7}\right] =$

20. Given the formula: $P = \dfrac{A}{1 + rt}$

Evaluate P if:

a. $A = 37{,}500$, $r = .0775$, $t = 14$

b. $A = 750$, $r = 0.125$, $t = 17$

21. Given the formula: $S = \dfrac{P}{A} + \dfrac{Mc}{I}$

Evaluate S if:

a. $P = 8400$, $A = 2.86$, $M = 6350$, $c = 3.20$, $I = 11.65$

b. $P = 12{,}500$, $A = 4.27$, $M = 7500$, $c = 4.62$, $I = 16.55$

22. Given the formula: $y_1 = \dfrac{I + Ay_0^2}{Ay_0}$

Evaluate y_1 if:

a. $I = 56.2$, $y_0 = 6.85$, $A = 19.4$

b. $I = 31.7$, $y_0 = 4.66$, $A = 12.7$

23. Given the formula: $\bar{x} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

Evaluate $\bar{x}$ if:

a. $m_1 = 2.6$, $x_1 = 1.7$, $m_2 = 3.5$, $x_2 = 4.5$, $m_3 = 5.2$, $x_3 = 7.7$

b. $m_1 = .0375$, $x_1 = 10.8$, $m_2 = .0466$, $x_2 = -5.05$, $m_3 = .0226$, $x_3 = 21.2$

24. Given the formula: $V_0 = \dfrac{VH}{760(1 + at)}$

Evaluate V_0 if:
a. $V = 270$, $H = 524$, $a = .00367$, $t = 62.5$
b. $V = 1500$, $H = 1340$, $a = .00367$, $t = 140$

Chapter 8

RATIO AND PROPORTION

8.1 Definitions

The *ratio* of a number M to another number N is the *quotient* M/N. Thus, the ratio of 4 to 3 refers simply to the quotient 4/3.

A *proportion* is a statement of *equality* between *two ratios*. The following represent examples of proportions:

$$\frac{3}{8} = \frac{9}{24}; \quad \frac{6}{4} = \frac{9}{6}; \quad \frac{x}{3} = \frac{2}{5}; \quad \frac{3.7}{6.5} = \frac{7.8}{y}$$

A statement of equality involving more than two ratios may be called a *continued proportion.* Following are examples of continued proportions:

$$\frac{3}{4} = \frac{6}{8} = \frac{18}{24}; \quad \frac{2}{3} = \frac{6}{x} = \frac{5}{y}; \quad \frac{6.1}{X} = \frac{Y}{2.8} = \frac{Z}{5} = \frac{7.3}{3.9}$$

It is often stated that M is *directly proportional* to N. This simply means that the *ratio* of M to N is *constant.*

8.2 Proportional settings on the C-D (CF-DF) scales

Suppose the slide is set so that the number M on the C scale is opposite the number N on the D scale. Then, for this position of the slide, the ratio of any other number on C to its opposite on D will be the same as the ratio of M to N. A similar relationship holds for the CF and DF scales.

Example 1: Find several ratios equivalent to 3/2.

1. Move HL over 2 on D.
2. Slide 3 on C under HL. The ratio 3/2 has now been set on the rule with the numerator on C opposite the denominator on D. Notice that 3 on CF is opposite 2 on DF; hence, opposite readings on the folded scales will also be in the ratio 3/2.
3. Move HL over 4 on C. Read "267" on D.
4. Move HL over 9 on C. Read "600" on D.
5. Move HL over 12 on CF. Read "800" on DF.

In this case, we have simply discovered the following continued proportion on the C-D (CF-DF) scales:

$$\frac{C(CF)}{D(DF)}: \frac{3}{2} = \frac{4}{2.67} = \frac{9}{6} = \frac{12}{8}$$

In the foregoing example, all the numerators appear on the C or CF scale, the denominators on the D or DF scale. Of course, the original ratio could have been set with the numerator on D and the denominator on C, in which case all the other numerators would be found on D (or DF) and denominators on C (or CF). In the following examples, the denominator will always be set on D (or DF).

Example 2: Solve the proportion:

$$\frac{2.7}{1.9} = \frac{x}{43}$$

This could be handled by first solving for x, and then performing the combined multiplication and division in the usual manner. However, in Figure 8.1 we illustrate how the proportion may be set up and solved directly on the slide rule without changing its form.

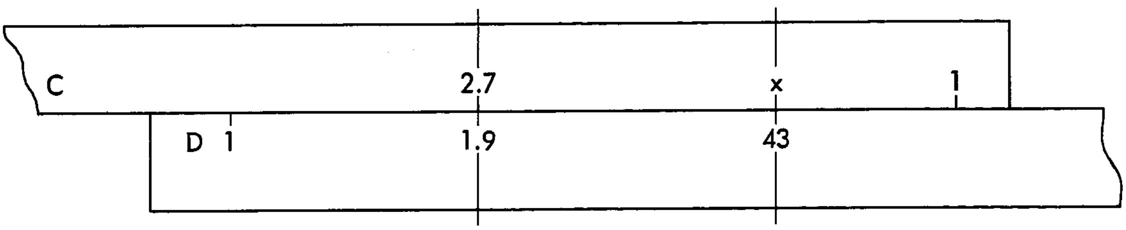

Figure 8.1

1. Move HL over 19 on D.
2. Slide 27 on C under HL. The known ratio is now set with numerator on C and denominator on D.
3. Move HL over 43 on D. Under HL read "611" on C.

Answer: $x =$ ***61.1.***

Example 3: Solve the following proportion on the C-D (CF-DF) scales.

$$\frac{C(CF)}{D(DF)}: \frac{45.1}{73} = \frac{2.2}{x} = \frac{y}{12.5}$$

1. Move HL over 73 on D.
2. Slide 451 on C under HL.
3. Move HL over 22 on C. Under HL read "356" on D.
4. Move HL over 125 on DF. Under HL read "772" on CF.

Answers: $x =$ ***3.56***, $y =$ ***7.72.***

Verify the following:

1. $\frac{215}{42} = \frac{y}{27}$; $y = 138.2$
2. $\frac{143}{R} = \frac{29.5}{6.75}$; $R = 32.7$
3. $\frac{71.5}{43.5} = \frac{124}{I}$; $I = 75.4$
4. $\frac{3.65}{5.20} = \frac{x}{22.4} = \frac{y}{0.63}$; $x = 15.72, y = 0.442$
5. $\frac{5.3}{V_1} = \frac{91}{44} = \frac{152}{V_2}$; $V_1 = 2.56, V_2 = 73.5$
6. $\frac{183}{34} = \frac{W}{16.4} = \frac{125}{L}$; $W = 88.2, L = 23.2$

Example 4: Solve the following proportion on the C-D (CF-DF) scales.

$$\frac{C\ (CF)}{D\ (DF)}: \frac{11.2}{8.63} = \frac{x}{47.5} = \frac{2.11}{y}$$

When the numerator and denominator of the given ratio are at opposite ends of the C and D scales, it is more convenient to set the ratio on the folded scales:

1. Move HL over 863 on DF.
2. Slide 112 on CF under HL.
3. Move HL over 475 on DF. Under HL read "616" on CF.
4. Move HL over 211 on CF. Under HL read "1627" on DF.

Answers: $x =$ ***61.6***, $y =$ ***1.627.***

Verify the following:

1. $\frac{125}{91} = \frac{x}{1.85}$; $x = 2.54$

2. $\frac{205}{x} = \frac{8.34}{1.16}$; $x = 28.5$

3. $\frac{545}{x} = \frac{y}{1.62} = \frac{13.2}{9.6}$; $x = 396, y = 2.23$

4. $\frac{x}{61} = \frac{113}{88} = \frac{125}{y}$; $x = 78.4, y = 97.3$

Example 5: $\frac{X}{.0275} = \frac{0.142}{56.2}$; $X = ?$

We may shift the decimal points three places to the left in the right-hand ratio, thus making its denominator comparable with the denominator of the left-hand ratio:

$$\frac{X}{.0275} = \frac{.000142}{.0562}$$

Since .0275 is about one half of .0562, X must be about one half of .000142. Verify that $X =$ ***.0000695.***

Example 6: $\frac{.0345}{162} = \frac{12.6}{X}$; $X = ?$

Shift decimal points in the left-hand ratio:

$$\frac{3.45}{16{,}200} = \frac{12.6}{X}$$

Since 12.6 is about four times 3.45, X must be about four times 16,200. Verify that $X =$ ***59,100.***

Exercise 8-1

Solve for the indicated letter or letters:

1. $\frac{X}{41.2} = \frac{3.84}{6.21}$

2. $\frac{X}{7.08} = \frac{21.2}{29.6}$

3. $\frac{X}{60.4} = \frac{179}{416}$

4. $\frac{4.16}{21.9} = \frac{X}{504}$

5. $\frac{3.14}{2.18} = \frac{X}{0.512}$

6. $\frac{X}{72.4} = \frac{3.18}{19.2}$

7. $\frac{T}{42.8} = \frac{12.2}{8.75}$

8. $\frac{254}{64.8} = \frac{T}{20.7}$

9. $\frac{9.60}{1.15} = \frac{39.6}{R}$

10. $\frac{5.74}{29.1} = \frac{8.68}{R}$

11. $\frac{41.6}{I} = \frac{105}{850}$

12. $\frac{14.25}{I} = \frac{.0253}{.00614}$

13. $\frac{546}{V} = \frac{182}{52.4}$

14. $\frac{.0362}{0.635} = \frac{7.54}{V}$

15. $\frac{194.5}{4.23} = \frac{39.6}{P}$

16. $\frac{24.3}{P} = \frac{.0053}{7.09}$

17. $\frac{X}{.0315} = \frac{0.219}{46.2}$

18. $\frac{1750}{X} = \frac{4.26}{.0615}$

19. $\frac{297}{71.4} = \frac{41.6}{L}$

20. $\frac{17.2}{283} = \frac{L}{147}$

21. $\frac{.00364}{W} = \frac{14.1}{22.6}$

22. $\frac{W}{5.63} = \frac{.0415}{.0022}$

23. $\frac{.0377}{12.75} = \frac{F}{6240}$

24. $\frac{.00452}{F} = \frac{0.724}{1255}$

25. $\frac{X}{3.54} = \frac{Y}{62.1} = \frac{370}{746}$

26. $\frac{X}{71.4} = \frac{Y}{2.07} = \frac{11.4}{9.10}$

27. $\frac{X}{0.824} = \frac{Y}{50.4} = \frac{12.55}{2.79}$

28. $\frac{856}{47.5} = \frac{X}{2.54} = \frac{Y}{5.21}$

29. $\frac{3.45}{23.7} = \frac{0.614}{V_1} = \frac{2.75}{V_2}$

30. $\frac{3.45}{x} = \frac{65.6}{5.12} = \frac{176}{y}$

31. $\frac{341}{76.1} = \frac{407}{V_1} = \frac{15.25}{V_2}$

32. $\frac{44.9}{V_1} = \frac{167}{V_2} = \frac{2.83}{5.49}$

33. $\frac{13.6}{74.5} = \frac{X}{6.24} = \frac{Y}{10.5} = \frac{4.20}{Z}$

34. $\frac{4.65}{X} = \frac{Y}{21.2} = \frac{14.25}{32.7} = \frac{Z}{7.60}$

35. $\frac{X}{90.4} = \frac{3.82}{Y} = \frac{16.38}{5.22} = \frac{640}{Z}$

36. $\frac{4.83}{2.11} = \frac{R_1}{.00482} = \frac{R_2}{0.204} = \frac{R_3}{0.651}$

37. $\frac{.0716}{0.477} = \frac{I_1}{38.2} = \frac{I_2}{275} = \frac{I_3}{1050}$

38. $\frac{1250}{83.5} = \frac{640}{T_1} = \frac{2670}{T_2} = \frac{135}{T_3}$

8.3 Conversion of units and other applications

Example 1: There are 16 ounces in a pound.
a. Convert to ounces: 4.3 lbs, 0.77 lbs.
b. Convert to pounds: 37.6 oz, 11.2 oz.

Here, the ratio of ounces to pounds is constant. Putting "ounces" on C opposite "pounds" on D, we may write:

$$\frac{\text{C: oz}}{\text{D: lbs}} = \text{constant;} \quad \text{(setting: } \textit{16 oz} \text{ opposite } \textit{1 lb}\text{)}$$

The continued proportion may be indicated in tabular form:

oz	C (CF)	**16**	?	?	37.6	11.2
lbs	D (DF)	**1**	4.3	0.77	?	?

1. Slide 16 on C opposite left index of D. This sets up the desired ratio on C-D.
2. Move HL over 43 on D. Under HL read "688" on C.
3. Move HL over 77 on DF. Under HL read "1232" on CF.
4. Move HL over 376 on C. Under HL read "235" on D.
5. Move HL over 112 on CF. Under HL read "700" on DF.

Answers are:
a. ***68.8 oz, 12.32 oz;*** **b.** ***2.35 lbs, 0.700 lbs.***

Example 2: 33 knots is equivalent to 38 mph.
a. Convert to mph: 40 knots, 6 knots.
b. Convert to knots: 10 mph, 25 mph.

Putting "knots" on C opposite "mph" on D:

$$\frac{\text{C: knots}}{\text{D: mph}} = \text{constant;} \quad \text{(setting: } \textit{33 knots} \text{ opposite } \textit{38 mph}\text{)}$$

knots	C (CF)	**33**	40	6	?	?
mph	D (DF)	**38**	?	?	10	25

1. Move HL over 38 on D. Slide 33 on C under HL. This sets up the ratio on C-D.
2. Verify the answers:
a. ***46.1 mph, 6.91 mph;*** **b.** ***8.69 knots, 21.7 knots.***

Example 3: On a map 1 inch is equivalent to 75 miles.
a. Convert to miles: 5.2 inches, 0.34 inches.
b. Convert to inches: 950 miles, 225 miles.

Putting "inches" on C opposite "miles" on D:

$$\frac{\text{C: inches}}{\text{D: miles}} = \text{constant;} \quad \text{(setting: } \textit{1 inch} \text{ opposite } \textit{75 miles}\text{)}$$

inches	C (CF)	**1**	5.2	0.34	?	?
miles	D (DF)	**75**	?	?	950	225

1. Set right index of C opposite 75 on D. This sets the constant ratio on C-D.
2. Verify the answers:
   a. ***390 miles, 25.5 miles***; b. ***12.67 inches, 3.00 inches.***

Example 4: A steel bar stretches .0073 inches under a load of 1200 lbs. If the stretch is directly proportional to the load, find the corresponding extensions of the bar for loads of 750 lbs, 2600 lbs, and 4250 lbs.

Inasmuch as this is a direct proportion, the ratio of stretch to load is constant. Putting "inches" on C opposite "lbs" on D:

$$\frac{\text{C: inches}}{\text{D: lbs}} = \text{constant};$$ (setting: *.0073 inches* opposite *1200 lbs*)

inches	C (CF)	**.0073**	?	?	?
lbs	D (DF)	**1200**	750	2600	4250

1. The slide will be in better position if the ratio is set on the *folded* scales. Move HL over 1200 on DF, slide 73 on CF under HL.
2. Verify the answers:
   .00456 inches, .0158 inches, and ***.0258 inches.***

Exercise 8-2

1. 53 liters is equivalent to 14 gallons.
   a. Convert to liters: 1 gal, 4.8 gal, 12.5 gal.
   b. Convert to gallons: 1 liter, 21 liters, 85 liters.

2. 30 mph is equivalent to 44 ft per sec.
   a. Convert to mph: 217 ft per sec, 68.2 ft per sec, 10 ft per sec.
   b. Convert to ft per sec: 48.2 mph, 21.7 mph, 85.6 mph.

3. 1 kilogram (kg) is equivalent to 2.2 lbs.
   a. Convert to lbs: 3.5 kg, 75.4 kg, 125 kg.
   b. Convert to kg: 34.6 lbs, 164 lbs, 4.75 lbs.

4. 14.7 lbs per sq in. is equivalent to 29.8 inches of mercury.
   a. Convert to inches of mercury: 21.2 lbs per sq in., 6.32 lbs per sq in., 135 lbs per sq in.
   b. Convert to lbs per sq in.: 0.723 inches mercury, 17.4 inches mercury.

5. On a map 1 inch is equivalent to 64 miles.
   a. Convert to miles: 2.4 in., 0.375 in., 4.75 in.
   b. Convert to inches: 150 miles, 275 miles.

6. 1 mile is equivalent to 1.61 kilometers (km).

a. Convert to km: 57.5 miles, 1245 miles, 845 miles.
b. Convert to miles: 452 km, 13.4 km.

7. 7.48 gals of water weighs 62.4 lbs.
a. Convert to lbs: 1 gal, 17.4 gals, 112 gals.
b. Convert to gals: 1940 lbs, 22.6 lbs.

8. 1 horsepower is equivalent to 746 watts.
a. Convert to hp: 124 watts, 36,500 watts, 1000 watts.
b. Convert to watts: 31.6 hp, 0.524 hp.

9. 31 square inches is approximately 200 square centimeters.
a. Convert to sq in: 350 sq cm, 62.5 sq cm, 756 sq cm.
b. Convert to sq cm: 148 sq in., 9.40 sq in.

10. In a circuit with constant resistance, the amperage (I) is proportional to the voltage (E). If $I = 8.74$ amps when $E = 120$ volts, find I corresponding to $E = 27.5$ volts, 110 volts, and 14.5 volts.

11. At constant velocity, distance is proportional to time. If it takes 10.5 hours to travel 645 miles, find the distance traveled in 1 hour, 6.25 hours, and 14.25 hours.

12. It requires 3.75 gallons of paint to cover 1450 square feet of surface. Assuming a direct proportion, find the amount of paint needed to cover 2100 sq ft, 6400 sq ft, 35,000 sq ft.

Chapter 9

SQUARES AND SQUARE ROOTS (A, B, R, SQ, AND $\sqrt{\ }$ SCALES)

9.1 Description of the A and B scales

The A and B scales are identical scales located on the body and slide respectively. Notice that each half of the A scale consists of a complete scale similar to D; thus, the A scale is made up of two half-length D scales, one next to the other. The two sections of the scale will be referred to as "A-left" and "A-right." Similar reference will be made to the B scale.

Since one complete A scale is half the length of the complete D scale, it follows that if the hairline is moved a certain distance along the D scale, it has effectively moved twice that distance relative to the A scale. For example, if the hairline is moved to the quarter point on D, it is at the half-way point on A. Inasmuch as these distances are proportional to the logarithms of the numbers, the reading on A must correspond to the *square* of the opposite reading on D. Conversely, the reading on D represents the *square root* of the opposite reading on A.

You can easily check this relationship by taking your slide rule and moving the hairline over "2" on D. Note that this puts the hairline over "4" on A-left. Now move the hairline over "3" on D; observe that hairline is over "9" on A-left. Finally, move the hairline over "5" on D and note that "25" is the opposite reading on A-right.

Summarizing:

> If the hairline is set over a number N on the **D** scale, then N^2 will be under the hairline on the **A** scale.
>
> Conversely, if the hairline is set over a number N on the *proper half* of the **A** scale, then $\sqrt{N}$ will be under the hairline on the **D** scale.

Because the A-B scales are half-length, you cannot read them as accurately as you read the C-D scales. Note especially that there are fewer subdivisions between primary numbers on the A-B scales.

9.2 The double-length scale (R, Sq, or $\sqrt{\ }$)

This scale appears with different labels on various commercial slide rules; depending on the make of the rule, you may find it identified as "R" (Frederick Post Co.), "Sq" (Keuffel & Esser Co.), or "$\sqrt{\ }$" (Pickett, Inc.) You will observe that the scale consists of two full-length sections, the first covers the same range as the left half of the D scale, whereas the second covers the same range as the right half of D. Thus, the two sections taken together represent one double-length D scale.

On the Post rule, these two sections are labeled R_1 and R_2 respectively; on the K & E rule they are denoted by Sq1 and Sq2. The Pickett model features the two sections of the $\sqrt{\ }$ scale "back-to-back"; that is, the two scales have a common axis, the upper markings corresponding to R_1 or Sq1, the lower markings corresponding to R_2 or Sq2. Hereafter, we shall simply use the R designation when referring to the double-length scale.

The R scale is related to the D scale in the same manner as the D scale is related to the A scale. Thus, the relationship may be described:

> If the hairline is set over a number N on the **R** scale, then N^2 will be under the hairline on the **D** scale.
>
> Conversely, if the hairline is set over a number N on the **D** scale, then $\sqrt{N}$ will be under the hairline on the *proper section* of the **R** scale.

The double-length scale may be read with greater accuracy than the A scale; hence, its advantage.

9.3 Squaring a number

Clearly, squaring a number may be handled as a multiplication in the conventional manner using the C (or CI) and D scales. However, with the A or R scales, squares may be obtained directly as illustrated in the following examples.

Example 1: $(2.48)^2 = ?$

Procedure with A scale:
1. Move HL over 248 on D.
2. Under HL read "615" on A.

Procedure with R scale:
1. Move HL over 248 on R_1.
2. Under HL read "615" on D.

Answer is ***6.15***.

Example 2: $(.00465)^2 = ?$

Write this: $(4.65 \times 10^{-3})^2 = (4.65)^2 \times 10^{-6}$.

Procedure with A scale:
1. Move HL over 465 on D.
2. Under HL read "216" on A.

Procedure with R scale:
1. Move HL over 465 on R_2.
2. Under HL read "216" on D.

Answer is ***21.6*** $\times$ ***10***$^{-6}$ or ***.0000216.***

Exercise 9-1

1. $(1.7)^2 =$

2. $(5.6)^2 =$

3. $(8.8)^2 =$

4. $(10.8)^2 =$

5. $(7.4)^2 =$

6. $(3.4)^2 =$

7. $(4.27)^2 =$

8. $(2.47)^2 =$

9. $(28.4)^2 =$

10. $(5.22)^2 =$

11. $(64.7)^2 =$

12. $(0.405)^2 =$

13. $(13.25)^2 =$

14. $(0.144)^2 =$

15. $(32.2)^2 =$

16. $(1.94 \times 10^2)^2 =$

17. $(2.77 \times 10^4)^2 =$

18. $(478)^2 =$

19. $(7840)^2 =$

20. $(56{,}000)^2 =$

21. $(162{,}500)^2 =$

22. $(4.05 \times 10^{-3})^2 =$

23. $(.0564)^2 =$

24. $(.00436)^2 =$

25. $(.000226)^2 =$

26. $(.0000362)^2 =$

27. $(18.65)^2 =$

28. $(56{,}400)^2 =$

29. $(207)^2 =$

30. $(456 \times 10^4)^2 =$

9.4 Finding square root of a number between 1 and 100

Example 1: $\sqrt{41.5} =$

Procedure with A scale:
Square roots are found by setting on A and reading on D. Note that the hairline may be

set over 415 on A-left or A-right. However, observe also that the desired square root is a number between 6 and 7; hence, it is clear that A-right is the proper choice.

1. Move HL over 415 on A-right.
2. Under HL read "644" on D. Answer is ***6.44.***

Procedure with R scale:
Square roots are found by setting on D and reading on R_1 or R_2. Here, we know the root is between 6 and 7; hence, answer must be on R_2.

1. Move HL over 415 on D.
2. Under HL read "644" on R_2. Answer is ***6.44.***

Example 2: $\sqrt{5.60} =$

By inspection, root is between 2 and 3.

Procedure with A scale:
1. Move HL over 560 on A-left.
2. Under HL read "237" on D.

Answer is ***2.37.***

Procedure with R scale:
1. Move HL over 560 on D.
2. Under HL read "2366" on R_1.

Answer is ***2.366.***

These examples illustrate the following rule:

1. When finding the square root of a number between *1 and 10,* use **A-left** or $\mathbf{R_1}$.
2. For numbers between *10 and 100,* use **A-right** or $\mathbf{R_2}$.

Again, you are reminded that square roots may also be obtained by reading from the proper half of the *B scale* to the *C scale.*

Verify the following:

1. $\sqrt{6.8} = 2.61$
2. $\sqrt{68} = 8.25$
3. $\sqrt{11.6} = 3.41$
4. $\sqrt{1.85} = 1.360$
5. $\sqrt{56.4} = 7.51$
6. $\sqrt{10} = 3.16$
7. $\sqrt{7.66} = 2.77$
8. $\sqrt{1.05} = 1.025$
9. $\sqrt{31.6} = 5.62$

9.5 A general procedure for square roots of numbers greater than 1

For numbers greater than 1, group the digits in *pairs* from right to left starting at the decimal point. Then:

1. If the leftmost group contains *one* digit, use **A-left** or **R_1**. If it contains *two* digits, use **A-right** or **R_2**.

2. The square root will contain the same number of *digits* to the *left* of the decimal point as there are *groups* to the *left* of the decimal point in the original number.

Example 1: $\sqrt{4{,}350{,}000} = ?$

1. Group the digits in pairs starting from the decimal point:

$$\sqrt{\underset{\smile}{4}\ \underset{\smile}{35}\ \underset{\smile}{00}\ \underset{\smile}{00}.}$$

↑ (leftmost group)

2. Leftmost group contains one digit; hence, use A-left or R_1.
3. Verify that slide rule reading is "209."
4. There are four groups to the left of the decimal point; therefore, answer will have four digits to left of decimal.

Answer is ***2090.***

Example 2: $\sqrt{167.5} = ?$

1. Group the digits in pairs starting from the decimal point:

$$\sqrt{\underset{\smile}{1}\ \underset{\smile}{67}.5}$$

↑ (leftmost group)

2. Leftmost group contains one digit; hence, use A-left or R_1.
3. Verify that slide rule reading is "1294."
4. There are two groups to the left of the decimal point; hence, answer will have two digits to left of decimal.

Answer is ***12.94.***

Example 3: $\sqrt{665{,}000} = ?$

1. Group the digits in pairs:

$$\sqrt{\underset{\smile}{66}\ \underset{\smile}{50}\ \underset{\smile}{00}.}$$

↑ (leftmost group)

2. Leftmost group contains two digits; hence, use A-right or R_2.
3. Verify that answer is ***815.***

Verify the following:

1. $\sqrt{352} = 18.76$	**4.** $\sqrt{615{,}000} = 784$	**7.** $\sqrt{1065} = 32.6$
2. $\sqrt{2090} = 45.7$	**5.** $\sqrt{1{,}265{,}000} = 1125$	**8.** $\sqrt{27{,}500} = 165.8$
3. $\sqrt{173.5} = 13.17$	**6.** $\sqrt{100{,}000} = 316$	**9.** $\sqrt{54{,}500{,}000} = 7380$

Example 4: $\sqrt[4]{3170} = ?$

We use the fact that the fourth root is the square root of the square root.

1. Verify that $\sqrt{3170} = 56.3$.
2. To find the fourth root, we extract the square root again:

$$\sqrt[4]{3170} = \sqrt{\sqrt{3170}} = \sqrt{56.3} = 7.50$$

On slide rules that have both the A scale and the double-length scale, the fourth root may be read directly. Thus, on such a rule, if the hairline is moved over a number N on the A scale, then $\sqrt[4]{N}$ will be under the hairline on the proper section of R (Sq or $\sqrt{\ }$).

Verify the following:

1. $\sqrt[4]{8.22} = 1.695$	**4.** $\sqrt[4]{16{,}500} = 11.35$
2. $\sqrt[4]{15.7} = 1.990$	**5.** $\sqrt[4]{5620} = 8.66$
3. $\sqrt[4]{315} = 4.21$	**6.** $\sqrt[4]{60{,}400{,}000} = 88.3$

9.6 General procedure for square roots of numbers less than 1

For numbers less than 1, group the digits in pairs from left to right starting at the decimal point. Then:

1. If the first nonzero group contains *one* significant digit, use **A-left** or $\mathbf{R_1}$. If it contains *two* significant digits, use **A-right** or $\mathbf{R_2}$.

2. For *each zero group* in the original number, the square root will have *one zero* immediately to the right of the decimal point.

Example 1: $\sqrt{.000617} = ?$

1. Group in pairs from left to right starting at decimal point:

$$\sqrt{.00\ 06\ 17}$$

(zero group) (first nonzero group)

2. First nonzero group contains *one* significant digit; hence, use *A-left* or R_1. Verify that slide rule reading is "248."

3. There is *one* zero group; therefore, answer will have *one* zero immediately after the decimal point. Result is ***.0248.***

Example 2: $\sqrt{.0000309} = ?$

1. Group in pairs from left to right:

$$\sqrt{.00\ 00\ 30\ 90}$$

(zero groups) (first nonzero group)

2. The first nonzero group contains *two* significant digits, hence use *A-right* or R_2. Verify that slide rule reading is "556."

3. There are *two* zero groups; therefore, answer will have *two* zeros immediately following the decimal point. Result is ***.00556.***

Example 3: $\sqrt{0.426} = ?$

1. Group in pairs from left to right:

$$\sqrt{0.42\ 60}$$

(first nonzero group)

2. First nonzero group contains *two* significant digits; hence, use *A-right* or R_2. Verify that slide rule reading is "653."

3. There are no zero groups; therefore, answer is ***0.653.***

Verify the following:

1. $\sqrt{.00475} = .0689$

2. $\sqrt{0.240} = 0.490$

3. $\sqrt{.0001425} = .01194$

4. $\sqrt{.0322} = 0.1794$

5. $\sqrt{.000061} = .00781$

6. $\sqrt{.00000845} = .00291$

7. $\sqrt[4]{.0050} = 0.266$

8. $\sqrt[4]{.0000008} = .0299$

9.7 Powers of 10 method

This method involves writing the original number in the form $M \times 10^n$, where M is a number between 1 and 100, and n is an *even* integer (positive or negative). The following examples illustrate:

Example 1: $\sqrt{430{,}000} = ?$

Rewrite as follows:

$$\sqrt{430{,}000} = \sqrt{43 \times 10^4} = \sqrt{43} \times 10^2 = \boldsymbol{6.56 \times 10^2}$$

Example 2: $\sqrt{.0000032} = ?$

Rewrite this:

$$\sqrt{.0000032} = \sqrt{3.2 \times 10^{-6}} = \sqrt{3.2} \times 10^{-3} = \boldsymbol{1.789 \times 10^{-3}}$$

Example 3: $\sqrt{2.25 \times 10^{-13}} = ?$

Rewrite:

$$\sqrt{2.25 \times 10^{-13}} = \sqrt{22.5 \times 10^{-14}} = \sqrt{22.5} \times 10^{-7} = \boldsymbol{4.74 \times 10^{-7}}$$

Example 4: $\sqrt{673 \times 10^7} = ?$

Rewrite this:

$$\sqrt{673 \times 10^7} = \sqrt{67.3 \times 10^8} = \sqrt{67.3} \times 10^4 = \boldsymbol{8.20 \times 10^4}$$

Again, you are reminded that the power of 10 must be *even;* that is, it must be *divisible by 2.*

Verify the following:

1. $\sqrt{6{,}100{,}000} = 2.47 \times 10^3$
2. $\sqrt{.00000055} = 7.42 \times 10^{-4}$
3. $\sqrt{2.75 \times 10^8} = 1.658 \times 10^4$
4. $\sqrt{375 \times 10^{-9}} = 6.12 \times 10^{-4}$
5. $\sqrt{14.2 \times 10^7} = 1.192 \times 10^4$
6. $\sqrt{10^{-5}} = 3.16 \times 10^{-3}$

Exercise 9-2

1. $\sqrt{48.6} =$
2. $\sqrt{5.43} =$
3. $\sqrt{12.65} =$
4. $\sqrt{1.06} =$
5. $\sqrt{59.8} =$
6. $\sqrt{10.8} =$
7. $\sqrt{4.78} =$
8. $\sqrt{364} =$
9. $\sqrt{3640} =$
10. $\sqrt{7460} =$
11. $\sqrt{992} =$
12. $\sqrt{19.65} =$

13. $\sqrt{\pi} =$

14. $\sqrt{27{,}400} =$

15. $\sqrt[4]{7200} =$

16. $\sqrt{31.4} =$

17. $\sqrt{0.764} =$

18. $\sqrt{0.415} =$

19. $\sqrt{.0423} =$

20. $\sqrt{.00177} =$

21. $\sqrt{.000717} =$

22. $\sqrt{656{,}000} =$

23. $\sqrt{.0000342} =$

24. $\sqrt[4]{.00643} =$

25. $\sqrt[4]{68{,}000} =$

26. $\sqrt{.000917} =$

27. $\sqrt{4.36 \times 10^{6}} =$

28. $\sqrt{2.43 \times 10^{-4}} =$

29. $\sqrt{5.64 \times 10^{-5}} =$

30. $\sqrt{27.4 \times 10^{-9}} =$

31. $\sqrt{2{,}060{,}000} =$

32. $\sqrt{.00060} =$

33. $\sqrt[4]{0.861} =$

34. $\sqrt{.0070} =$

35. $\sqrt{3.14 \times 10^{-3}} =$

36. $\sqrt{.0001925} =$

37. $\sqrt{37{,}000{,}000} =$

38. $\sqrt{2.17 \times 10^{-7}} =$

39. $\sqrt[4]{283} =$

40. $\sqrt{.000211} =$

41. $\sqrt{2.72 \times 10^{6}} =$

42. $\sqrt{2.18 \times 10^{-6}} =$

43. $\sqrt{8.45} =$

44. $\sqrt[4]{.00529} =$

45. $\sqrt{.000816} =$

46. $\sqrt{2.07 \times 10^{11}} =$

47. $\sqrt{5.64 \times 10^{7}} =$

48. $\sqrt{28{,}700} =$

49. $\sqrt[4]{0.385} =$

50. $\sqrt{5.82 \times 10^{5}} =$

9.8 Formula types

Example 1: $\sqrt{3 + \dfrac{112}{16.2}} = ?$

1. Verify that $\dfrac{112}{16.2} = 6.91$.

2. Expression may now be evaluated:

$$\sqrt{3 + 6.91} = \sqrt{9.91} = \boldsymbol{3.15}.$$

As illustrated in the next examples, it is sometimes better to change the form of an expression before evaluating.

Example 2: $\pi[(4.15)^2 - (4.12)^2] = ?$

To find the difference in squares which are of about the same magnitude, more accuracy is obtained by factoring first:

$$\pi[(4.15)^2 - (4.12)^2] = \pi(4.15 + 4.12)(4.15 - 4.12) = \pi(8.27)(.03)$$

Verify that the result is ***0.780.***

Example 3: $\dfrac{\sqrt{14} - \sqrt{13}}{2} = ?$

Here, we must find a small difference in square roots, and more accuracy is obtained if we first multiply and divide by $(\sqrt{14} + \sqrt{13})$:

$$\frac{\sqrt{14} - \sqrt{13}}{2} = \frac{(\sqrt{14} - \sqrt{13})(\sqrt{14} + \sqrt{13})}{2(\sqrt{14} + \sqrt{13})} = \frac{14 - 13}{2(\sqrt{14} + \sqrt{13})} = \frac{1}{2(\sqrt{14} + \sqrt{13})}$$

Verify that:

$$\frac{1}{2(\sqrt{14} + \sqrt{13})} = \frac{1}{2(3.74 + 3.61)} = \frac{1}{2(7.35)} = \textbf{\textit{.0680.}}$$

Exercise 9-3

1. $\sqrt{2 + \dfrac{16.3}{4.2}} =$

2. $\sqrt{28 - \dfrac{46}{3.5}} =$

3. $\pi[(3.22)^2 - (3.19)^2] =$

4. $\pi[(236)^2 - (234)^2] =$

5. $\sqrt{\dfrac{1}{22} + \dfrac{1}{44}} =$

6. $\sqrt{\dfrac{1}{2.5} + \dfrac{1}{3.5} + \dfrac{1}{5.5}} =$

7. $\dfrac{\sqrt{230} - \sqrt{228}}{2} =$

8. $\dfrac{\sqrt{31.2} - \sqrt{30.9}}{.034} =$

9. $\sqrt{\dfrac{36.7}{2.33} + \dfrac{165}{7.26}} =$

10. $\sqrt{\dfrac{53.7}{72.2} - (0.72)^2} =$

11. $\dfrac{2.65}{1 - (0.985)^2} =$

12. $\dfrac{1}{\sqrt{118} - \sqrt{116}} =$

13. $\dfrac{32.2}{\sqrt{12.8} - \sqrt{12.5}} =$

14. $\dfrac{31.6 + \sqrt{153}}{12.6} =$

15. $\dfrac{.0866 - \sqrt{.00324}}{2.77} =$

16. $\dfrac{13 + \sqrt{(13)^2 + (4 \times 12 \times 21)}}{2 \times 12} =$

17. $\dfrac{-6.82 + \sqrt{(6.82)^2 - (4 \times 2.33 \times 1.52)}}{2 \times 2.33} =$

18. $\sqrt{(2.6)^2 + (4.7)^2 + (5.2)^2} =$

19. $\dfrac{\sqrt{17.8} - \sqrt{6.22}}{0.27 \times 3.32} =$

20. $\dfrac{\sqrt{.0535} + \sqrt{0.136}}{.0064 \times 5.22} =$

In the following formulas, substitute as indicated and evaluate:

21. $T = \sqrt{a + \dfrac{1}{a}}$

a. $a = 1.65$
b. $a = 2.23$
c. $a = 0.644$

22. $t = \dfrac{KA}{A_0} (h_1^{1/2} - h_2^{1/2})$

a. $K = 12.6$, $A = 13.6$, $A_0 = 0.74$, $h_1 = 7$, $h_2 = 5$
b. $K = 22.8$, $A = 7.45$, $A_0 = 0.330$, $h_1 = 14$, $h_2 = 6$
c. $K = 7.25$, $A = 25.6$, $A_0 = 2.74$, $h_1 = 131$, $h_2 = 29$

23. $R = \dfrac{k + \sqrt[4]{1 + r^2}}{16.1}$

a. $k = 18.2$, $r = 1.77$
b. $k = 5.66$, $r = 0.785$
c. $k = 34.7$, $r = 6.55$

24. $h = \dfrac{\sqrt{p_0} + \sqrt{p_1}}{p_0 p_1}$

a. $p_0 = 7.65$, $p_1 = 1.66$
b. $p_0 = 5.20$, $p_1 = 28.6$
c. $p_0 = .0440$, $p_1 = 0.655$

25. $d = \sqrt{a^2 + b^2 + c^2}$

a. $a = 2.2$, $b = 3.6$, $c = 4.7$
b. $a = 13.7$, $b = 2.27$, $c = 8.66$

9.9 The quadratic formula

The two roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve for x: $2.84x^2 + 5.35x - 3.75 = 0$.

1. In this case, $a = 2.84$, $b = 5.35$, and $c = -3.75$. Substituting these values, the formula becomes:

$$x = \frac{-5.35 \pm \sqrt{(5.35)^2 - 4(2.84)(-3.75)}}{2(2.84)}$$

2. Verify that $x = \dfrac{-5.35 \pm \sqrt{28.6 + 42.5}}{5.68} = \dfrac{-5.35 \pm 8.44}{5.68}$

3. Finally verify that the two roots are ***0.544*** and ***−2.43.***

Exercise 9-4

Solve for x:

1. $2x^2 + 3x - 1 = 0$

2. $5x^2 - 17x + 6 = 0$

3. $7x^2 - 13x - 5 = 0$

4. $16x^2 + x - 0.30 = 0$

5. $x^2 - 11.7x + 4.55 = 0$

6. $2.65x^2 + 6.82x + 1.70 = 0$

7. $31.2x^2 + 23.7x + 2.18 = 0$

8. $0.810x^2 + 17.6x - 31.2 = 0$

Chapter 10

CUBES AND CUBE ROOTS (K AND $\sqrt[3]{}$ SCALES)

10.1 Description of the K scale*

The K scale consists of three complete scales, one next to the other, each similar to the D scale. Thus, each section represents a one-third-length D scale. We shall refer to these sections as "K-left," "K-middle," and "K-right." Because of the logarithmic nature of the scales, it follows that the one-third-length K scale leads to a cube-cube root relationship with the D scale. To verify this, suppose you now take your slide rule and move the hairline over "2" on the D scale. Note that this positions the hairline over "8" on K-left. Now move the hairline over "3" on D; note that this puts the hairline over "27" on K-middle. Finally, move the hairline over "5" on D and observe that the hairline is over "125" on K-right. In each case, the reading on K is the *cube* of the corresponding reading on D.

Summarizing:

> If the hairline is set over a number N on the **D** scale, then N^3 will be under the hairline on the **K** scale.
>
> Conversely, if the hairline is set over a number N on the *proper section* of the **K** scale, then $\sqrt[3]{N}$ will be under the hairline on the **D** scale.

In making settings and readings on the K scale, carefully note how the primary intervals are subdivided. Clearly, the use of this scale involves considerable loss in accuracy; over much of the scale you will find yourself straining to approximate the third digit.

*If your slide rule does not have a K scale, but does have a scale labeled $\sqrt[3]{}$, then refer to section 10.7 of this chapter.

10.2 Cubing a number using the K scale

Of course, cubing a number may be treated as successive multiplication using the C, CI, and D scales, thus retaining the accuracy of these scales. However, as indicated in the previous section, cubes may be read directly from D to K.

Example 1: $(3.45)^3 = ?$

1. Move HL over 345 on D.
2. Under HL read "410" on K-middle. Answer is ***41.0.***

Example 2: $(645)^3 = ?$

Write this: $(6.45 \times 10^2)^3 = (6.45)^3 \times 10^6$.

1. Move HL over 645 on D.
2. Under HL read "268" on K-right.

Answer is 268×10^6 or $\boldsymbol{2.68 \times 10^8}$.

Example 3: $(.00172)^3 = ?$

Write this: $(1.72 \times 10^{-3})^3 = (1.72)^3 \times 10^{-9}$.

1. Move HL over 172 on D.
2. Under HL read "509" on K-left. Answer is $\boldsymbol{5.09 \times 10^{-9}}$.

Exercise 10-1

1. $(1.6)^3 =$
2. $(2.4)^3 =$
3. $(6.5)^3 =$
4. $(7.2)^3 =$
5. $(1.23)^3 =$
6. $(5.82)^3 =$
7. $(3.24)^3 =$
8. $(2.76)^3 =$
9. $(8.04)^3 =$
10. $(0.764)^3 =$
11. $(0.512)^3 =$
12. $(10.65)^3 =$
13. $(12.7)^3 =$
14. $(8180)^3 =$
15. $(29{,}600)^3 =$
16. $(364)^3 =$
17. $(135)^3 =$
18. $(0.907)^3 =$
19. $(1265)^3 =$
20. $(4.65 \times 10^4)^3 =$

21. $(0.787)^3 =$

22. $(0.215)^3 =$

23. $(3.96 \times 10^{-3})^3 =$

24. $(0.404)^3 =$

25. $(.0643)^3 =$

26. $(.00284)^3 =$

27. $(.00847)^3 =$

28. $(17{,}550)^3 =$

29. $(68.2)^3 =$

30. $(.000545)^3 =$

10.3 Finding cube root of a number between 1 and 1000

As stated before, if the hairline is set over a number on the proper section of the K scale, the cube root of that number will be under the hairline on the D scale. You can easily verify the following rule for selecting the proper section of the K scale:

1. For cube roots of numbers between *1 and 10,* use ***K-left.***
2. For numbers between *10 and 100,* use ***K-middle.***
3. For numbers between *100 and 1000,* use ***K-right.***

Example 1: $\sqrt[3]{3.60} = ?$

1. The number is between 1 and 10; hence, move HL over 360 on K-left.
2. Under HL read "1532" on D. Answer is ***1.532.***

Example 2: $\sqrt[3]{45.4} = ?$

1. The number is between 10 and 100; hence, move HL over 454 on K-middle.
2. Under HL read "357" on D. Answer is ***3.57.***

Example 3: $\sqrt[3]{720} = ?$

1. The number is between 100 and 1000; hence, move HL over 720 on K-right.
2. Under HL read "896" on D. Answer is ***8.96.***

Verify the following:

1. $\sqrt[3]{7.4} = 1.950$ **2.** $\sqrt[3]{32} = 3.17$ **3.** $\sqrt[3]{615} = 8.50$

4. $\sqrt[3]{12.1} = 2.30$ **6.** $\sqrt[3]{10} = 2.15$ **8.** $\sqrt[3]{75} = 4.22$

5. $\sqrt[3]{4.25} = 1.620$ **7.** $\sqrt[3]{100} = 4.64$ **9.** $\sqrt[3]{134} = 5.12$

10.4 General procedure for cube roots

In order to determine which section of the K scale to use for any number, the digits may be grouped in a manner similar to that used for square roots. However, instead of marking the digits off in pairs, they are marked off in groups of *three.* The following rule then applies:

For numbers greater than 1:

1. If the leftmost group contains *one* digit, use ***K-left;*** if it contains *two* digits, use ***K-middle;*** if it contains *three* digits, use ***K-right.***

2. The cube root will contain the same number of *digits to the left* of the decimal point as there are *groups to the left* of the decimal point in the original number.

For numbers less than 1:

1. If the first nonzero group contains *one* significant digit, use ***K-left;*** if it contains *two* significant digits, use ***K-middle;*** if it contains *three* significant digits, use ***K-right.***

2. For *each zero group* in the original number, the cube root will contain *one zero* between the decimal point and the first significant figure.

Example 1: $\sqrt[3]{43{,}500} = ?$

1. Grouping the digits: $\sqrt[3]{\underbrace{43}\ \underbrace{500}}$
2. Leftmost group contains *two* digits; hence, use *K-middle.* Verify that the slide rule reading is "352."
3. There are *two* groups; hence, answer contains *two* digits to the left of the decimal point. Result is ***35.2.***

Example 2: $\sqrt[3]{2{,}140{,}000} = ?$

1. Grouping the digits: $\sqrt[3]{\underbrace{2}\ \underbrace{140}\ \underbrace{000}}$.
2. Leftmost group contains *one* digit; hence, use *K-left.* Verify that slide rule reading is "1288."
3. There are *three* groups; therefore, answer is ***128.8.***

Example 3: $\sqrt[3]{.000720} = ?$

1. Grouping the digits: $\sqrt[3]{.000\ 720}$
2. First nonzero group contains *three* significant digits; hence, use *K-right*. Slide rule reading is "896."
3. There is *one zero group;* therefore, there is *one zero* between the decimal point and the first significant digit. Answer is ***.0896.***

Example 4: $\sqrt[3]{.00564} = ?$

1. Grouping the digits: $\sqrt[3]{.005\ 640}$
2. First nonzero group contains *one* significant digit; hence, use *K-left*. Slide rule reading is "178."
3. There is *no* zero group; therefore, answer is ***0.178.***

Verify the following:

1. $\sqrt[3]{6400} = 18.57$
2. $\sqrt[3]{.0215} = 0.278$
3. $\sqrt[3]{365{,}000} = 71.5$
4. $\sqrt[3]{0.820} = 0.936$
5. $\sqrt[3]{.000100} = .0464$
6. $\sqrt[3]{.0000048} = .01687$
7. $\sqrt[3]{62{,}000{,}000} = 396$
8. $\sqrt[3]{.00000075} = .00909$
9. $\sqrt[3]{4{,}250{,}000} = 162$
10. $\sqrt[3]{.0325} = 0.319$

10.5 Powers of 10 method

Using this method, the original number is written in the form $M \times 10^n$, where M is a number between 1 and 1000, and n is a positive or negative integer which is *divisible by three.* The following examples illustrate:

Example 1: $\sqrt[3]{56{,}100{,}000} = ?$

Rewrite: $\sqrt[3]{56.1 \times 10^6} = \sqrt[3]{56.1} \times 10^2 = 3.83 \times 10^2 = $ ***383.***

Example 2: $\sqrt[3]{.000000750} = ?$

Rewrite: $\sqrt[3]{750 \times 10^{-9}} = \sqrt[3]{750} \times 10^{-3} = 9.09 \times 10^{-3} = $ ***.00909.***

Example 3: $\sqrt[3]{4.66 \times 10^{10}} = ?$

Rewrite: $\sqrt[3]{46.6 \times 10^9} = \sqrt[3]{46.6} \times 10^3 = 3.60 \times 10^3 = $ ***3600.***

Example 4: $\sqrt[3]{2.34 \times 10^{-10}} = ?$

Rewrite: $\sqrt[3]{234 \times 10^{-12}} = \sqrt[3]{234} \times 10^{-4} = 6.16 \times 10^{-4} =$ ***.000616.***

Again, you are reminded that the power of 10 must be *divisible by three.*

Verify the following:

1. $\sqrt[3]{1.84 \times 10^{7}} = 264$
2. $\sqrt[3]{6.47 \times 10^{-13}} = 8.65 \times 10^{-5}$
3. $\sqrt[3]{10^{8}} = 464$
4. $\sqrt[3]{10^{-11}} = .000216$
5. $\sqrt[3]{482 \times 10^{16}} = 1.69 \times 10^{6}$
6. $\sqrt[3]{42 \times 10^{-7}} = .01613$
7. $\sqrt[3]{36.5 \times 10^{8}} = 1540$
8. $\sqrt[3]{7.25 \times 10^{11}} = 8980$

Exercise 10-2

1. $\sqrt[3]{7.43} =$
2. $\sqrt[3]{8.22} =$
3. $\sqrt[3]{10.9} =$
4. $\sqrt[3]{96.2} =$
5. $\sqrt[3]{17.65} =$
6. $\sqrt[3]{124} =$
7. $\sqrt[3]{742} =$
8. $\sqrt[3]{2.94} =$
9. $\sqrt[3]{294} =$
10. $\sqrt[3]{57.4} =$
11. $\sqrt[3]{1.11} =$
12. $\sqrt[3]{11.1} =$
13. $\sqrt[3]{111} =$
14. $\sqrt[3]{.0624} =$
15. $\sqrt[3]{0.526} =$
16. $\sqrt[3]{0.912} =$
17. $\sqrt[3]{.0207} =$
18. $\sqrt[3]{.0611} =$
19. $\sqrt[3]{.0829} =$
20. $\sqrt[3]{.00299} =$
21. $\sqrt[3]{.00748} =$
22. $\sqrt[3]{3920} =$
23. $\sqrt[3]{4680} =$
24. $\sqrt[3]{9600} =$
25. $\sqrt[3]{2000} =$
26. $\sqrt[3]{7640} =$
27. $\sqrt[3]{10,700} =$
28. $\sqrt[3]{23,600} =$
29. $\sqrt[3]{64,300} =$
30. $\sqrt[3]{575,000} =$

31. $\sqrt[3]{207{,}000} =$

32. $\sqrt[3]{87{,}500} =$

33. $\sqrt[3]{.00726} =$

34. $\sqrt[3]{.000276} =$

35. $\sqrt[3]{.0000427} =$

36. $\sqrt[3]{23{,}600{,}000} =$

37. $\sqrt[3]{.00000717} =$

38. $\sqrt[3]{180{,}000{,}000} =$

39. $\sqrt[3]{.0000007} =$

40. $\sqrt[3]{76.1 \times 10^6} =$

41. $\sqrt[3]{2.76 \times 10^{-6}} =$

42. $\sqrt[3]{4.68 \times 10^{-5}} =$

43. $\sqrt[3]{5.64 \times 10^{-2}} =$

44. $\sqrt[3]{11.6 \times 10^{-4}} =$

45. $\sqrt[3]{1.95 \times 10^5} =$

46. $\sqrt[3]{12.65 \times 10^8} =$

47. $\sqrt[3]{0.416 \times 10^{10}} =$

48. $\sqrt[3]{4.26 \times 10^{-10}} =$

49. $\sqrt[3]{7.61 \times 10^{14}} =$

50. $\sqrt[3]{3.37 \times 10^{-11}} =$

10.6 Finding $(N)^{3/2}$ and $(N)^{2/3}$

The general problem of raising a number to any power is discussed later in connection with the log-log scales. However, certain fractional powers may be read directly using the A and K scales.

Example 1: $(56.2)^{3/2} = (\sqrt{56.2})^3 = ?$

1. Move HL over 562 on A-right. Note that $\sqrt{56.2}$ is now under HL on D, and $(\sqrt{56.2})^3$ is under HL on K.
2. Under HL read "421" on K. Answer is ***421.***

If you do not have the A scale, you may first evaluate $(56.2)^3$, then find the square root of this on R.

Example 2: $(7.2)^{2/3} = (\sqrt[3]{7.2})^2 = ?$

1. Move HL over 72 on K-left. Observe that $\sqrt[3]{7.2}$ is now under HL on D, and $(\sqrt[3]{7.2})^2$ is under HL on A.
2. Under HL read "373" on A. Answer is ***3.73.***

If no A scale is present, you may first find $\sqrt[3]{7.2}$, then multiply this by itself.

These examples illustrate the following relationship:

1. If HL is set over a number N on the **A** scale, then $(N)^{3/2}$ will be under HL on the **K** scale.

2. If HL is set over a number N on the **K** scale, then $(N)^{2/3}$ will be under HL on the **A** scale.

Example 3: $(2.75 \times 10^{13})^{2/3} = ?$

Write this: $(27.5 \times 10^{12})^{2/3} = (27.5)^{2/3} \times 10^{8}$.

1. Move HL over 275 on K-middle.
2. Under HL read "910" on A. Answer is $\boldsymbol{9.10 \times 10^{8}}$.

Exercise 10-3

1. $(4.5)^{3/2} =$	11. $(342)^{2/3} =$
2. $(37)^{3/2} =$	12. $(1.65)^{2/3} =$
3. $(16.3)^{3/2} =$	13. $(2340)^{2/3} =$
4. $(0.814)^{3/2} =$	14. $(.0621)^{2/3} =$
5. $(.0614)^{3/2} =$	15. $(.000175)^{2/3} =$
6. $(.00743)^{3/2} =$	16. $(0.445)^{3/2} =$
7. $(374)^{3/2} =$	17. $(3.22 \times 10^{-9})^{3/2} =$
8. $(.000514)^{3/2} =$	18. $(7.45 \times 10^{8})^{2/3} =$
9. $(2460)^{3/2} =$	19. $(63 \times 10^{-7})^{2/3} =$
10. $(46.2)^{2/3} =$	20. $(5.75 \times 10^{9})^{3/2} =$

10.7 The triple-length scale ($\sqrt[3]{\ }$)

Some Pickett models feature a $\sqrt[3]{\ }$ scale consisting of three full-length sections; the first covers the same range as the first third of the regular D scale (left index to "2154"), the second covers the range of the middle third of D ("2154 to "464"), and the third corresponds to the final third of the D scale ("464" to right index). We shall refer to these sections as $(\sqrt[3]{\ })_1$, $(\sqrt[3]{\ })_2$, and $(\sqrt[3]{\ })_3$ respectively. Clearly, the three sections taken together represent one triple-length D scale.

The $\sqrt[3]{\ }$ scale is related to the D scale as follows:

> If the hairline is set over a number N on the $\sqrt[3]{\ }$ scale, then N^3 will be under the hairline on the **D** scale.
>
> Conversely, if the hairline is set over a number N on the **D** scale, then $\sqrt[3]{N}$ will be under the hairline on the *proper section* of the $\sqrt[3]{\ }$ scale.

Example 1: $(3.47)^3 = ?$

1. Move HL over 347 on $(\sqrt[3]{\ })_2$.
2. Under HL read "418" on D. Answer is ***41.8.***

Example 2: $\sqrt[3]{6450} = ?$

To determine which section of the $\sqrt[3]{\ }$ scale applies, you should refer to sections 10.3 and 10.4 of this chapter—substituting $(\sqrt[3]{\ })_1$ for K-left, $(\sqrt[3]{\ })_2$ for K-middle, and $(\sqrt[3]{\ })_3$ for K-right.

1. Grouping the digits: $\sqrt[3]{\underset{\smile}{6}\ \underset{\smile}{450}}$.
2. Leftmost group contains *1 digit;* hence, answer is on $(\sqrt[3]{\ })_1$.
3. Move HL over 645 on D. Under HL read "1861" on $(\sqrt[3]{\ })_1$.
4. There are *two groups;* hence, there are *two digits* to the left of the decimal point in the answer. Result is ***18.61.***

Example 3: $\sqrt[3]{.000224} = ?$

1. Grouping the digits: $\sqrt[3]{.\underset{\smile}{000}\ \underset{\smile}{224}}$.
2. The first nonzero group contains *three* significant digits, hence answer is on $(\sqrt[3]{\ })_3$.
3. Move HL over 224 on D. Under HL read "607" on $(\sqrt[3]{\ })_3$.
4. There is *one zero group;* therefore, answer has *one zero* immediately following the decimal point. Result is ***.0607.***

Exercises 10-1 and 10-2 may be used for more drill with these scales.

Chapter 11

COMBINED OPERATIONS WITH SQUARES

11.1 The A and B scales as operational scales

We have seen how the A scale may be used with the D scale to find squares and square roots. However, inasmuch as the A and B scales are identical (one fixed and one movable), they may also be used as operational scales in the same manner as the C and D; that is, multiplication, division, and combinations of these operations may be carried out entirely on the A and B scales.

Example 1: $23.6 \times 1.85 = ?$ (Use A and B scales)

1. Set left index of B opposite 236 on A-left. (Note that middle index of B is also opposite 236 on A-right).
2. Move HL over 185 on B-left.
3. Under HL read "436" on A-left. Answer is ***43.6.***

Observe that hairline could have been moved over 185 on B-right, and the result read on A-right.

Example 2: $\dfrac{126 \times 0.83}{4.6} = ?$ (Use A and B scales)

1. Move HL over 126 on A-right. This is more centrally located on the rule than 126 on A-left.
2. Slide 46 on B-left under HL. (We could just as well choose B-right).
3. Move HL over 83 on B-left.
4. Under HL read "227" on A-right. Answer is ***22.7.***

Example 3: $\frac{56}{3.7\times 2.9} = ?$ (Use A and B scales)

1. Move HL over 56 on A-left.
2. Slide 37 on B-left under HL. Before we can divide again, it is necessary to move HL over B index.
3. Move HL over middle B index. Slide 29 on B-right under HL.
4. Opposite right index of B, read "522" on A-right. Answer is ***5.22***.

Example 4: $\frac{1.2}{2.7} = \frac{x}{4.8} = \frac{7.6}{y}$; $x = ?, y = ?$

1. Move HL over 12 on A-left.
2. Slide 27 on B-left under HL.
3. Move HL over 48 on B-left (or right). Under HL read "213" on A-left (or right).
4. Move HL over 76 on A-left. Under HL read "171" on B-right.

Answers: $x =$ ***2.13***, $y =$ ***17.1***.

Clearly, the A-B scales are especially convenient for proportions; however, the disadvantage is that they cannot be read as accurately as the C and D scales. Also, most rules do not have a reciprocal B scale which reduces efficiency in combined operations. However, we shall see that, for combined operations involving the square or square root, the A-B scales can be used effectively in conjunction with the C and D scales.

Verify the following (use A-B scales):

1. $23 \times 4.6 = 106$
2. $\frac{56}{2.1} = 26.7$
3. $\frac{3.2 \times 7.1}{2.8} = 8.11$
4. $1.7 \times 3.8 \times 2.6 = 16.8$
5. $\frac{740}{26 \times 15} = 1.90$
6. $\frac{3.7}{7.1} = \frac{X}{2.6} = \frac{4.5}{Y}$ ($X = 1.355$; $Y = 8.63$)

11.2 Simple operations with squares

It is clear that any combined operation involving squared quantities may be evaluated in the conventional manner, the squares being treated simply as multiplication. In this section, however, we wish to show how the A-B or R scales may be used when the square is involved.

Example 1: $2.6 \times (5.4)^2 = ?$

Procedure with A-B scales:

1. Set right index of C opposite 54 on D. Note that $(5.4)^2$ is now on A opposite right index of B; hence, remaining multiplication by 2.6 may be carried out on the A-B scales.

2. Move HL over 26 on B.
3. Under HL read "758" on A. Answer is ***75.8.***

Alternatively, you may first set B index opposite 26 on A, then move hairline over 54 on C and read the result on A. Also, you could first move the hairline over 26 on A, then slide 54 on CI under hairline with the result appearing opposite B index on A.

Procedure with the R scale:

1. Move HL over 54 on R_2. HL is now over $(5.4)^2$ on D, and we may continue on the C-D scales.
2. Slide 26 on CI under HL. This multiplies by 2.6 on C-D.
3. Opposite right index of C read "758" on D. Answer is ***75.8.***

Example 2: $\frac{(6.75)^2}{3.5} = ?$

Procedure with A-B scales:

1. Move HL over 675 on D. HL is now over $(6.75)^2$ on A; hence, division by 3.5 may be carried out on the A-B scales.
2. Slide 35 on B-right under HL. (Observe that 35 on B-left could also be moved under HL; however, choosing B-right leaves slide in better position).
3. Opposite B index (either left or middle index) read "130" on A. Answer is ***13.0.***

Procedure with R scale:

1. Move HL over 675 on R_2. HL is now over $(6.75)^2$ on D; hence, division by 3.5 may be carried out on the C-D scales.
2. Slide 35 on C under HL.
3. Opposite left index of C, read "1302" on D. Answer is ***13.02.***

Example 3: $\frac{64.5}{(2.14)^2} = ?$

Procedure with A-B scales:

1. Move HL over 645 on A-left.
2. Slide 214 on C under HL. This positions $(2.14)^2$ on B under HL; thus, we have divided 64.5 by $(2.14)^2$ on the A-B scales.
3. Opposite left (or middle) index of B, read "141" on A.

Answer is ***14.1.***

Procedure with R scale:
The R scale is not convenient when the squared quantity appears in the denominator. We may simply treat the square as a product, and evaluate on C-D in the usual manner.

Verify that: $\frac{64.5}{(2.14)^2} = \frac{64.5}{2.14 \times 2.14} = \boldsymbol{14.08}.$

Observe in the foregoing examples that numbers to be squared are set on C-D, unsquared numbers are set on A-B, and the actual operation takes place on the A-B scales. If the R scale is used, a number to be squared is set on R (if it occurs in the numerator), unsquared numbers are set on C-D, and the actual operation takes place on the C-D scales.

Verify the following:

1. $3.4 \times (2.7)^2 = 24.8$

2. $\frac{(3.8)^2}{4.6} = 3.14$

3. $\frac{(27.2)^2}{64.5} = 11.5$

4. $1.65 \times (4.77)^2 = 37.5$

5. $\frac{5.7}{(1.8)^2} = 1.76$

6. $\frac{473}{(9.25)^2} = 5.52$

11.3 Area of a circle

The area of a circle is given by either of the two formulas:

$$A = \pi r^2 \quad \text{(where } r = \text{radius)}$$
$$A = \frac{\pi}{4} d^2 \quad \text{(where } d = \text{diameter)}$$

Both π and $\pi/4$ are usually marked for you on the A and B scales. The factor $\pi/4$ is approximately equal to 0.785 and, on most rules, you will find a scribed mark at this location on A-right and B-right.

Example 1: Find areas of circles with diameters equal to 2.45 in., 4.22 in., and 6.40 in., respectively.

1. Move HL over scribed mark at $\pi/4$ on A-right.
2. Slide right index of B under HL. You are now set up to multiply this factor by d^2. If the HL is moved over d on the C scale, d^2 will be under the HL on B and the desired area will be under the HL on A.
3. Move HL over 245 on C. Under HL read "471" on A.
4. Move HL over 422 on C. Under HL read "140" on A.
5. Move HL over 640 on C. Under HL read "321" on A.

Answers are: ***4.71*, *14.0*,** and ***32.1*** sq. in., respectively.

Example 2: Find areas of circles with radii equal to 1.45 cm., 2.24 cm., and 4.06 cm., respectively.

Procedure with A-B scales:
1. Move HL over scribed mark at π on A-left.
2. Slide left index of B under HL. You are now set up to multiply this factor by r^2. If the HL is moved over r on the C scale, r^2 will be under HL on B and the desired product will be under HL on A.

3. Move HL over 145 on C. Under HL read "660" on A.
4. Move HL over 224 on C. Under HL read "158" on A.
5. Move HL over 406 on C. Under HL read "518" on A.

Answers are: ***6.60, 15.8,*** and ***51.8*** sq. cm., respectively.

Procedure with R scale:
If the HL is set over r on the R scale, then r^2 will be under HL on D, and πr^2 will be under HL on DF. Therefore, if r is set on R, the area is read directly on DF.

1. Move HL over 145 on R_1. Under HL read "660" on DF.
2. Move HL over 224 on R_1. Under HL read "1576" on DF.
3. Move HL over 406 on R_2. Under HL read "518" on DF.

Answers are: ***6.60, 15.76,*** and ***51.8*** sq. cm. respectively.

This last example exhibits a convenient property of the R scale:

> *To find the area of a circle:*
> 1. Move HL over *radius* on R.
>
> 2. Under HL read *area* on DF.

Exercise 11-1

1. $3.24 \times (4.25)^2 =$

2. $1.68 \times (2.75)^2 =$ "127"

3. $21.4 \times (3.75)^2 =$

4. $12.6 \times (5.06)^2 =$ "323"

5. $4.63 \times (16.2)^2 =$

6. $64.3 \times (0.463)^2 =$ "138"

7. $.0723 \times (21.6)^2 =$

8. $\dfrac{(4.67)^2}{3.26} =$ "670"

9. $\dfrac{(26.7)^2}{19.2} =$

10. $\dfrac{(36.5)^2}{127} =$ "105"

11. $\dfrac{(11.45)^2}{16.2} =$

12. $\dfrac{4.65}{(1.67)^2} =$ "167"

13. $\dfrac{29.6}{(2.37)^2} =$

14. $\dfrac{347}{(14.2)^2} =$ "172"

15. $\dfrac{1275}{(21.6)^2} =$

16. $\dfrac{45.6}{(0.843)^2} =$ "642"

17. $\dfrac{(264)^2}{5.65} =$

18. $\dfrac{(515)^2}{28.5}$ = "930"

19. $230 \times (0.86)^2 =$

20. $0.76 \times (22.5)^2$ = "385"

21. $\dfrac{7.44}{(.00362)^2} =$

22. $\dfrac{(1560)^2 \times 10^{-4}}{42.5}$ = "573"

23. $\dfrac{3.75 \times 10^{-7}}{(560)^2} =$

24. Find areas of circles with following radii:

a. 25.1 in. **b.** 5.82 in. **c.** 0.377 ft. **d.** 53.1 ft. **e.** .0104 cm.

25. Find areas of circles with following diameters:

a. 2.68 in. **b.** 1.77 in. **c.** 4.65 ft. **d.** 7.05 ft. **e.** 14.65 ft.

26. Evaluate $y = 2.84\,x^2$ for $x = 2, 3, 5, 7$, and 9.

27. Evaluate $s = 16.1\,t^2$ for $t = 1.5, 2.5, 3.5$, and 5.5.

28. Evaluate $P = .074\,I^2$ for $I = 2, 5, 12, 25$, and 60.

In the following exercises, use the CI scale in conjunction with A-B:

29. Evaluate $y = 14.7/x^2$ for $x = 3.4, 5.5, 7.5$, and 12.6.

30. Evaluate $y = 375/x^2$ for $x = 11, 15, 19$, and 25.

31. Evaluate $h = 180/r^2$ for $r = 2.2, 4.6, 7.5$, and 11.5.

11.4 Further operations with squares

Example 1: $\dfrac{(7.2)^2}{2.4 \times (3.1)^2} = ?$

Procedure with A-B scales:

1. Move HL over 72 on D. HL is now over $(7.2)^2$ on A.
2. Slide 24 on B-right under HL. This divides by 2.4 on A-B.
3. Move HL over 31 on CI. This positions HL over $1/(3.1)^2$ on B; therefore, we have multiplied by the reciprocal of $(3.1)^2$ on A-B.
4. Under HL read "225" on A. Answer is ***2.25.***

Procedure with R scale:

1. Move HL over 72 on R_2. HL is now over $(7.2)^2$ on D.
2. Slide 31 on C under HL. This divides by 3.1.
3. Move HL over 31 on CI. This divides again by 3.1.
4. Slide 24 on C under HL. This divides by 2.4.
5. Opposite left index of C, read "225" on D. Answer is ***2.25.***

Example 2: $$\frac{(2.7)^2 \times (4.6)^2}{6.3} = ?$$

Procedure with A-B scales:

1. Move HL over 27 on D.
2. Slide 63 on B-left under HL.
3. Move HL over 46 on C.
4. Under HL read "245" on A. Answer is ***24.5.***

Procedure with R scale:

1. Move HL over 27 on R_1.
2. Slide 63 on C under HL.
3. Move HL over 46 on C.
4. Slide 46 on CI under HL.
5. Opposite left index of C, read "245" on D. Answer is ***24.5.***

Example 3: $$\frac{64}{(3.1 \times 2.2)^2} = \frac{64}{(3.1)^2 \times (2.2)^2} = ?$$

Procedure with A-B scales:

1. Move HL over 64 on A-left.
2. Slide 31 on C under HL.
3. Move HL over 22 on CI.
4. Under HL read "138" on A. Answer is ***1.38.***

Procedure with R scale:

Since the squares occur in the denominator, the R scale is not convenient. This may be evaluated as a combined operation in the conventional manner, treating the squares as products. Verify that the result is ***1.377.***

Example 4: $$\left[\frac{2.7 \times 4.1 \times 6.3}{7.2 \times 3.5}\right]^2 = ?$$

Procedure with A-B scales:

In this case, all the settings are made on C-D and the result appears on the A scale. Verify that the answer is ***7.65.***

Procedure with R scale:

When several factors are to be squared, you may first evaluate the expression to be squared on the C-D scales. This result may then be multiplied by itself on C-D, or it may be transferred via hairline to the R scale with the answer appearing under the hairline on D. Verify that the result is ***7.65.***

Verify the following:

1. $$\frac{(7.5)^2}{1.2 \times (3.7)^2} = 3.42$$

2. $$\frac{(15.2 \times 0.72)^2}{36.3} = 3.30$$

3. $\dfrac{(26.4)^2}{\pi \times (19.3)^2} = 0.596$

4. $\dfrac{675}{(5.66)^2 \times 7.23} = 2.92$

5. $\dfrac{184}{(6.2 \times 5.6)^2} = 0.153$

6. $\left[\dfrac{6.34 \times 37.5}{1.23 \times 42.2}\right]^2 = 20.9$

Example 5: $\dfrac{(2.3)^3}{1.7} = ?$

Write: $\dfrac{(2.3)^3}{1.7} = \dfrac{(2.3)^2 \times 2.3}{1.7}$

Procedure with A-B scales:

1. Move HL over 23 on D. Operation now continues on A-B:
2. Slide 17 on B under HL.
3. Move HL over 23 on B.
4. Under HL read "715" on A. Answer is ***7.15.***

Procedure with R scale:

1. Move HL over 23 on R_1. Operation now continues on C-D:
2. Slide 17 on C under HL.
3. Move HL over 23 on C.
4. Under HL read "715" on D. Answer is ***7.15.***

If your slide rule has a $\sqrt[3]{\ }$ scale, you may obtain $(2.3)^3$ directly on D with a single setting; then simply divide by 1.7.

Example 6: $(2.85)^4 = ?$

Write: $(2.85)^4 = (2.85)^2 \times (2.85)^2$.

Procedure with A-B scales:

1. Set left index of C opposite 285 on D.
2. Move HL over 285 on C.
3. Under HL read "660" on A. Answer is ***66.0.***

Procedure with R scale:

1. Move HL over 285 on R_1. Observe that HL is over 812 on D.
2. Slide 812 on CI under HL.
3. Opposite left index of C read "660" on D.

Answer is ***66.0.***

Note that fourth powers may be read *directly* on slide rules that have both the A scale and the double-length scale. Thus, on such rules, if the hairline is set over a number N on R (Sq or $\sqrt{\ }$), N^4 will be under the hairline on A.

Example 7: $(0.82)^5 = ?$

Integral powers such as this arise frequently in probability calculations.

Verify that $(0.82)^5 = (0.82)^2 \times (0.82)^2 \times 0.82 =$ ***0.370.***

The techniques illustrated by the foregoing examples may be summarized as follows:

Combined Operations with Squares

Procedure with A-B scales:

1. Numbers to be squared are set on D, C, or CI, thus locating the squares on A or B.

2. Unsquared quantities are set directly on A or B, and the actual operation takes place on the A-B scales.

Procedure with R scale:

If a squared number is to be further multiplied or divided:

1. Number to be squared is set on R, thus locating its square on D.

2. Operation then continues on C-D; if more squares are involved, they are treated as products.

Exercise 11-2

1. $\dfrac{(2.6)^2 \times (5.3)^2}{45} =$

2. $\dfrac{(3.4)^2 \times (6.1)^2}{58} =$ "742"

3. $\dfrac{(3.4 \times 37.2)^2}{63.4} =$

4. $\dfrac{(2.7 \times 8.6)^2}{23} =$ "234"

5. $\dfrac{45}{(3.8)^2 \times (5.2)^2} =$

6. $\dfrac{28}{(2.6)^2 \times (1.8)^2} =$ "128"

7. $(3.17)^4 =$

8. $\dfrac{(1.56)^4}{3.40} =$ "174"

9. $\dfrac{73.2}{(2.15)^4} =$

10. $\dfrac{(5.2)^3}{4.6} =$ "305"

11. $\dfrac{(3.8)^3}{(2.6)^2} =$

12. $(2.32)^5 =$ "672"

13. $\dfrac{370}{(4.25 \times 0.660)^2} =$

14. $\dfrac{57.3}{(12.8 \times 0.455)^2} =$ "169"

15. $(2.44 \times 3.65 \times 5.22)^2 =$

16. $(2.65 \times 1.82 \times 3.22 \times 0.72)^2 =$ "125"

17. $(3.52 \times 4.62)^2 \times 0.711 =$

18. $(0.524)^2 \times (21.5)^2 \times 3.14 =$ "398"

19. $\dfrac{(3.78)^2 \times 14.2}{2.64} =$

20. $\dfrac{(5.27)^2 \times 34.6}{62.7} =$ "153"

21. $\pi \times (4.6)^2 \times 7.2 =$

22. $\pi \times (15.4)^2 \times 12.7 =$ "946"

23. $\left(\frac{94.3}{43.1}\right)^2 =$

24. $\left(\frac{52.4}{2.94}\right)^2 =$ "318"

25. $\left(\frac{28.2}{3.82 \times 2.64}\right)^2 =$

26. $\left(\frac{36.1}{4.20 \times 1.75}\right)^2 =$ "241"

27. $\frac{(1640 \times .0264)^2}{14.65} =$

28. $\frac{(64.3)^2 \times .0143}{(11.4)^2} =$ "455"

29. $\frac{(4.72)^4}{3.14 \times 10^6} =$

30. $\frac{(.0463)^2 \times 154}{(.0706)^2} =$ "661"

31. $\frac{(1.87)^3}{4.05} =$

32. $\frac{(21.4)^2}{(3.75)^3} =$ "869"

33. $\frac{(12.6)^3}{(5.42)^2} =$

34. $\frac{1}{36.3} \times \left(\frac{127}{4.21}\right)^2 =$ "251"

35. $\left(\frac{28.7 \times .0824}{.00365}\right)^2 =$

36. $\left(\frac{2.75 \times 3200}{1.64 \times .0346}\right)^2 =$ "240"

37. $(0.83)^4 =$

38. $(0.42)^4 =$ "311"

39. $(0.155)^5 =$

40. $(0.845)^5 =$ "430"

41. $(0.72)^6 =$

42. $(0.28)^6 =$ "481"

43. $(5.4)^5 =$

44. $(13)^7 =$ "628"

11.5 Formula types

Example 1: $\frac{19}{1.3} \times \left(\frac{1}{4.2} + \frac{1}{2.3}\right)^2 = ?$

1. Use CI or DI scale to verify that $\frac{1}{4.2} + \frac{1}{2.3} = 0.238 + 0.435 = 0.673$

2. Expression may now be evaluated as:

$$\frac{19 \times (0.673)^2}{1.3} = \mathbf{6.61}.$$

Example 2: Given the formula: $I = \frac{\pi}{32}(D_1^4 - D_2^4)$.

Find I when $D_1 = 4.25$ and $D_2 = 3.85$.

1. Substituting the given values:

$$I = \frac{\pi}{32}\,[(4.25)^4 - (3.85)^4].$$

2. Verify that $(4.25)^4 = 327$, and $(3.85)^4 = 220$.
3. Expression may now be evaluated:

$$I = \frac{\pi}{32}\,(327 - 220) = \frac{\pi}{32} \times 107 = \boldsymbol{10.5}.$$

Exercise 11-3

1. $32.4\,(1 + 16/3.7)^2 =$

2. $\dfrac{15000}{1 + \dfrac{(109)^2}{15000}} =$

3. $\dfrac{2.74 \times (3.66)^2}{(8.22)^2 - (3.66)^2} =$

4. $\dfrac{45.3 \times (13.5)^2}{(27.6)^2 - (13.5)^2} =$

5. $\dfrac{(216 - 212.4)^2}{212.4} + \dfrac{(115 - 107.6)^2}{107.6} + \dfrac{(64 - 61.2)^2}{61.2} =$

6. $\dfrac{(21.8 - 19.4)^2}{19.4} + \dfrac{(16.3 - 13.5)^2}{13.5} + \dfrac{(8.82 - 7.16)^2}{7.16} =$

7. $62.4\,[7.22 \times 8.13 - \pi\,(4.35)^2] =$

8. $\sqrt[3]{\dfrac{(2.6)^3(1.7) + (3.4)^3(2.2)}{1.7 + 2.2}} =$

9. $\sqrt[3]{\dfrac{(5.74)^3(2.66) + (7.22)^3(3.72)}{2.66 + 3.72}} =$

10. $\dfrac{64.5}{4.66}\left[\dfrac{1}{6.32} + \dfrac{1}{4.25}\right]^2 =$

11. $\dfrac{350}{52.4}\left[\dfrac{1}{3.66} + \dfrac{1}{7.25}\right]^2 =$

12. $\dfrac{(\sqrt{73} + \sqrt{6.4})^2}{\pi} =$

13. $\dfrac{(\sqrt{235} + \sqrt{61.2})^2}{\pi} =$

In the following formulas, substitute as indicated and evaluate:

14. $p = p_0 + \dfrac{1}{2} \rho V_0^2$

a. $p_0 = 275$, $\rho = .00238$, $V_0 = 124$
b. $p_0 = 1340$, $\rho = .00238$, $V_0 = 365$

15. $I = \dfrac{\pi D^4}{128}$

a. $D = 3.44$
b. $D = 6.32$

16. $\Theta = \dfrac{32\ TL}{\pi G\,(D_1^4 - D_2^4)}$

a. $T = 11{,}300$, $G = 12 \times 10^6$, $D_1 = 2.75$, $D_2 = 2.25$, $L = 5.50$
b. $T = 7500$, $G = 12 \times 10^6$, $D_1 = 1.85$, $D_2 = 1.65$, $L = 12.6$

17. $d = \dfrac{5wL^4}{384EI}$

a. $I = 168$, $w = 21.5$, $L = 115$, $E = 1.3 \times 10^6$
b. $I = 255$, $w = 36.2$, $L = 76$, $E = 1.3 \times 10^6$

18. $P = 15p^2q^4$

a. $p = 0.46$, $q = 0.54$
b. $p = 0.28$, $q = 0.72$

19. $P = 56\,p^5q^3$

a. $p = 0.42$, $q = 0.58$
b. $p = 0.86$, $q = 0.14$

20. $P = \dfrac{n!}{x!\,(n-x)!}\,p^x q^{n-x}$

a. $n = 7$, $x = 2$, $p = 0.12$, $q = 0.88$
b. $n = 7$, $x = 3$, $p = 0.33$, $q = 0.67$

Chapter 12

COMBINED OPERATIONS WITH ROOTS

12.1 Simple operations involving the square root

In this section we consider certain combined operations with the square root. Here again, the A-B scales may be used to advantage in conjunction with the C-D scales. The following examples illustrate the procedures with the A-B scales, and also with the R scale.

Example 1: $27 \times \sqrt{4.5} = ?$

Procedure with A-B scales:

1. Move HL over 45 on A-left. HL is now over $\sqrt{4.5}$ on D, and we are now in a position to multiply by 27 on the C-D scales.
2. Slide 27 on CI under HL. This multiplies by 27.
3. Opposite right index of C, read "573" on D. Answer is ***57.3***.

Note that in step (1) we are not free to choose either section of the A scale. We are locating the square root of 4.5 on D; therefore, we must select the proper section of A in accordance with the rules for square root.

Procedure with R scale:
You may find $\sqrt{4.5}$ on R, transfer the result to D, and then multiply by 27. An alternate procedure which avoids the transfer operation is the following:

1. Think of the given expression as $\sqrt{(27)^2 \times 4.5}$.
2. Move HL over 27 on R_1; this puts $(27)^2$ on D. Now multiply by 4.5:
3. Slide right index of C under HL; move HL over 45 on C. The result of this multiplication is now under HL on D, and the square root of this result is under the HL on R_1 or R_2. A rough estimate indicates that the answer must be on R_2.
4. Under HL read "573" on R_2. Answer is ***57.3***.

Example 2: $\frac{\sqrt{50.5}}{2.35} = ?$

Procedure with A-B scales:

1. Move HL over 505 on A-right. HL is now over $\sqrt{50.5}$ on D; therefore, the division is carried out on the C-D scales:
2. Slide 235 on C under HL.
3. Opposite left index of C, read "302" on D. Answer is ***3.02.***

Again, in step (1), we must choose the proper section of A.

Procedure with R scale:
Here, you may find $\sqrt{50.5}$ on R, transfer the result to D, and divide by 2.35.
Alternatively, you may think of the expression as $\sqrt{50.5/(2.35)^2}$. The steps would then be:

1. Move HL over 505 on D. Now divide by 2.35:
2. Slide 235 on C under HL. Now divide again by 2.35:
3. Move HL over 235 on CI. Square root is now under HL on R_1 or R_2. A rough estimate indicates that the result is on R_1.
4. Under HL read "3022" on R_1. Answer is ***3.022.***

Verify the following:

1. $\sqrt{21} \times 3.6 = 16.5$
2. $\sqrt{6.3} \times 12.7 = 31.9$
3. $0.45 \times \sqrt{140} = 5.33$
4. $\frac{\sqrt{3.2}}{1.4} = 1.278$
5. $\frac{\sqrt{43.0}}{5.70} = 1.150$
6. $14.2 \times \sqrt{39.1} = 88.8$
7. $243 \times \sqrt{.0241} = 37.7$
8. $37.5 \times \sqrt{.00425} = 2.44$
9. $\frac{\sqrt{12.4}}{4.73} = 0.745$
10. $\frac{\sqrt{5750}}{8.61} = 8.81$

Example 3: $\frac{18}{\sqrt{29}} = ?$

Procedure with A-B scales:

1. Move HL over 18 on D.
2. Slide 29 on B-right under HL. Note that $\sqrt{29}$ is now under HL on C, hence, the division has been performed on the C-D scales.
3. Opposite right index of C, read "334" on D. Answer is ***3.34.***

Procedure with R scale:
You may first find $\sqrt{29}$ on R, make a mental note of this (or write it down), move HL over 18 on D, and then complete the division. Alternatively, the expression may be evaluated as $\sqrt{(18)^2/29}$: