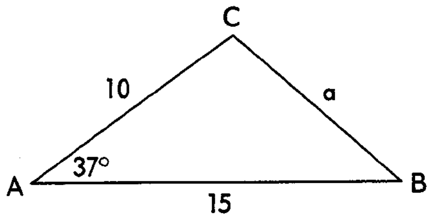

Figure 17.6

1. Use law of cosines to find side a:

$$a^2 = 10^2 + 15^2 - 2(10)(15)\cos 37°$$
$$a = \sqrt{100 + 225 - 300 \cos 37°}$$

Verify that $a = \mathbf{9.24}$.

2. Now apply law of sines:

$$\frac{\sin 37°}{9.24} = \frac{\sin B}{10} = \frac{\sin C}{15}$$

Solve for angle B first; it is the smaller angle, and must be acute. Verify that $B = \mathbf{40.7°}$.

3. Finally, $C = 180° - (40.7° + 37°) = \mathbf{102.3°}$.

Note that $\sin 102.3° = \sin(180° - 102.3°) = \sin 77.7°$, and verify that 77.7° satisfies the proportion; that is, 77.7° is opposite 15.

Example 2: Given $B = 126°$, $a = 13$, $c = 22$. Find remaining parts (Figure 17.7).

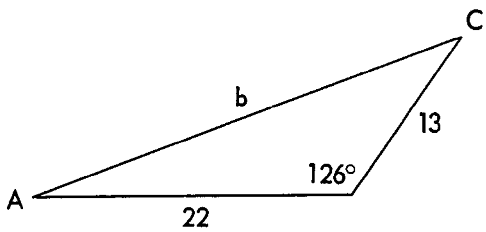

Figure 17.7

1. Use law of cosines to find side b:

$$b^2 = (13)^2 + (22)^2 - 2(13)(22)\cos 126°.$$

But $\cos 126° = -\cos(180° - 126°) = -\cos 54°$; hence, we write:

$$b = \sqrt{(13)^2 + (22)^2 + 2(13)(22)\cos 54°}$$

Verify that $b = \mathbf{31.5}$.

2. Apply law of sines:

$$\frac{\sin 126°}{31.5} = \frac{\sin A}{13} = \frac{\sin C}{22}$$

Both A and C must be acute. Verify that $A = \mathbf{19.5°}$, $C = \mathbf{34.5°}$.

Example 3: Given $a = 6.56$, $b = 4.30$, $c = 3.70$. Find remaining parts.

1. We use the law of cosines to find one of the acute angles; say, angle C:

$$\cos C = \frac{(6.56)^2 + (4.30)^2 - (3.70)^2}{2(6.56)(4.30)}$$

Verify that $\cos C = 0.848$, from which $C = \mathbf{32°}$.

2. Apply law of sines:

$$\frac{\sin A}{6.56} = \frac{\sin B}{4.30} = \frac{\sin 32°}{3.70}$$

Now solve for B first (which must also be acute), and then A. Verify that $B = \mathbf{38°}$ and $A = \mathbf{110°}$.

Exercise 17-3

Solve the triangles.

1. $a = 5$, $b = 7$, $C = 38°$

2. $b = 12$, $c = 17$, $A = 49°$

3. $a = 6$, $b = 8$, $c = 11$

4. $a = 12$, $b = 13$, $c = 13$

5. $a = 6$, $c = 11$, $B = 118°$

6. $C = 112.5°$, $a = 10$, $b = 15$

7. $a = 9$, $b = 4$, $c = 12$

8. $a = 11$, $b = 12$, $c = 17$

9. $a = 21.6$, $c = 36.4$, $B = 17.2°$

10. $a = 6.57$, $c = 14.15$, $B = 124.6°$

11. $a = 35$, $b = 29$, $c = 52$

12. $a = 4.75$, $b = 15.6$, $c = 17.25$

13. $a = 7.45$, $c = 3.62$, $B = 104.6°$

14. $A = 5°$, $b = 80$, $c = 60$

15. $a = 245$, $b = 280$, $c = 312$

16. $a = 56.4$, $b = 61.7$, $c = 111$

Chapter 18

LOGARITHMS TO BASE 10 (L SCALE)

18.1 Description of the L scale

The L scale is a uniform scale increasing from 0 to 1. The primary division marks are usually labeled 0, .1, .2, .3, and so forth; however, on some slide rules the decimal point is omitted. Whether shown or not, the decimal point is understood to be there. Depending upon the model, the L scale will be found either on the body of the rule, or on the slide.

18.2 Finding logarithms to base 10

In the following sections, if the base is omitted it is understood that the base 10 is intended; thus "log N" means "$\log_{10} N$."

We recall that, when using tables to find log N, the table gives us the mantissa only; the characteristic is determined by inspection. It will be seen that the L scale takes the place of the table; that is, it supplies the mantissa.

If the L scale is on the body of the rule, it is related to the D scale; if it is on the slide, it is related to the C scale. The relationship may be stated:

To find log N: 1. Move HL over N on **D** scale (or **C** scale if L is on slide). 2. Under HL read *mantissa* of log N on the **L** scale. 3. Determine the characteristic by inspection, and add it to the mantissa for the complete log.

Example 1: log 36.4 = ?

1. Move HL over 364 on D (or C if L is on slide).
2. Under HL read 0.561 on L. This is the mantissa.

Characteristic is 1; hence, log 36.4 = ***1.561.***

Example 2: log 10,850 = ?

1. Move HL over 1085 on D (or C if L is on slide).
2. Under HL read 0.035 on L. This is the mantissa.

Characteristic is 4; hence, log 10,850 = ***4.035.***

Example 3: log .00784 = ?

1. Move HL over 784 on D (or C if L is on slide).
2. Under HL read 0.894 on L. This is the mantissa.

Characteristic is −3; hence, log .00784 = ***7.894 − 10.*** The result could also be presented as the negative number: ***−2.106.***

Example 4: $10^x = 52.7$; $x = ?$

Here, $x = \log 52.7$. Verify that $x =$ ***1.722.***

Verify the following:

1. log 17.5 = 1.243
2. log 524 = 2.719
3. log 1225 = 3.088
4. log .0738 = 8.868 − 10
5. log .00064 = −3.194
6. log 2.07 = 0.316
7. $10^x = 34{,}500$; $x = 4.538$
8. $10^x = .0124$; $x = -1.907$

18.3 Finding the antilogarithm

Here, we are concerned with the reverse problem: given log *N*, find *N*. This is called finding the antilogarithm of a number; thus "antilog *x*" means "the number whose log is *x*."

Example 1: antilog 3.497 = ?

The mantissa is 0.497; therefore, proceed as follows:

1. Move HL over 0.497 on L.
2. Under HL read "314" on D (or C if L is on slide).

The characteristic is 3; hence, result is ***3140.***

Example 2: $\log N = 9.022 - 10$; $N = ?$

1. Move HL over 0.022 on L.
2. Under HL read "1052" on D (or C if L is on slide).

Characteristic is −1; therefore, $N =$ ***0.1052.***

Example 3: antilog −1.375 = ?

In order to discover the mantissa and characteristic, we add and subtract 10:

$$-1.375 = (-1.375 + 10) - 10 = 8.625 - 10$$

We now see that the mantissa is 0.625 and the characteristic is −2.

1. Move HL over 0.625 on L.
2. Under HL read "422" on D (or C if L is on slide).

Result is ***.0422.***

Verify the following:

1. antilog 1.328 = 21.3
2. antilog 3.031 = 1074
3. antilog (8.444 − 10) = .0278
4. antilog −2.397 = .00401
5. antilog 8.064 = 1.159×10^8
6. antilog −18.943 = 1.14×10^{-19}

18.4 Finding 10^x

By definition, $10^x = \text{antilog } x$.

Example: $10^{1.45} = ?$

We merely find antilog 1.45:

1. Move HL over 0.45 on L.
2. Under HL read "282" on D (or C if L is on slide).

Characteristic is 1; hence, $10^{1.45} =$ ***28.2.***

Verify the following:

1. $10^{2.78} = 603$

2. $10^{.037} = 1.089$

3. $10^{4.853} = 71{,}300$

4. $10^{-2.974} = .001062$

Exercise 18-1

1. log 345 =

2. log 108 =

3. log 11.6 =

4. log .063 =

5. log .00545 =

6. log 425,000 =

7. log 2070 =

8. log 0.320 =

9. log .000185 =

10. log (4.23×10^{7}) =

11. log (2.03×10^{12}) =

12. log (1.77×10^{-9}) =

13. log (6.34×10^{-21}) =

14. $10^{x} = 27.5$; x =

15. $10^{x} = .00175$; x =

16. $10^{-x} = .0633$; x =

17. $10^{-2x} = 550$; x =

18. antilog 2.466 =

19. antilog (9.006 − 10) =

20. antilog (−1.264) =

21. antilog (−0.076) =

22. $10^{3.022}$ =

23. $10^{0.315}$ =

24. $10^{-1.267}$ =

25. $10^{-2.705}$ =

26. $10^{15.075}$ =

27. $10^{-12.716}$ =

28. log $N = -1.436$; N =

29. log $N = 9.825$; N =

30. log $N = -11.307$; N =

18.5 Formula types

Example 1: In chemistry, the pH value of a solution is related to the hydrogen ion concentration (H^+) as follows:
$pH = -\log(H^+)$.

a. Find pH value if $(H^+) = .00450$.
1. Substituting in the formula: $pH = -\log .00450$.
2. Verify that $pH = -(7.653 - 10) =$ ***2.347.***

b. Find (H^+) if $pH = 6.150$.
1. Substituting: $6.150 = -\log(H^+)$.
2. Solve for $\log(H^+)$: $\log(H^+) = -6.150 = 3.850 - 10$.
3. Verify that $(H^+) = \mathbf{7.08 \times 10^{-7}}$.

Verify the following:

1. $(H^+) = .000325$; $pH = 3.488$
2. $(H^+) = .0644$; $pH = 1.191$
3. $(H^+) = 1.035 \times 10^{-8}$; $pH = 7.985$
4. $pH = 3.123$; $(H^+) = .000753$
5. $pH = 1.624$; $(H^+) = .0238$
6. $pH = 9.045$; $(H^+) = 9.02 \times 10^{-10}$

Example 2: $\frac{1}{\pi}\log\left(\frac{156}{342}\right) = ?$

1. Divide 156 by 342 on C-D, and use indicator to read the corresponding log on L. Verify that the log thus obtained is $9.659 - 10 = -0.341$.
2. Move HL over 341 on DF. Under HL read "1085" on D.

Result is ***−0.1085.***

Example 3: Given the formula: $C = \dfrac{0.1208\,L}{\log\left(\frac{2D}{d}\right) - \frac{D^2}{8h^2}}$

Evaluate C when $L = 365$, $D = 4.25$, $d = 0.275$, $h = 6.3$.

1. Verify that $\log\left(\frac{2D}{d}\right) = \log\frac{2 \times 4.25}{0.275} = 1.49.$
2. Verify that $\frac{D^2}{8h^2} = \frac{(4.25)^2}{8(6.3)^2} = .0569.$
3. C may now be evaluated:

$$C = \frac{0.1208 \times 365}{1.49 - .0569} \approx \frac{0.1208 \times 365}{1.433} = \mathbf{30.7.}$$

Exercise 18-2

1. $3.12 \log 275 =$
2. $6.3 \log\left(\frac{1.84}{0.31}\right) =$
3. $\pi \log\left(\frac{26.3}{4.77}\right) =$
4. $(2.84)^2 \log\left(\frac{4.65}{175}\right) =$
5. $\frac{37.2 \log .00633}{2.05} =$
6. $\frac{2.72 \log 14.7}{\log 2800} =$

7. $\dfrac{\log .0664}{13.5 \log 7.23} =$

8. $(\log 655)^2 =$

9. $.045\sqrt{3.58 + \log 263} =$

10. $0.575 \times 10^{3.075} =$

11. $7.66 \times 10^{-9.622} =$

12. $(H^+) = .000109$; pH =

13. $(H^+) = 5.66 \times 10^{-10}$; pH =

14. $(H^+) = 1.074 \times 10^{-9}$; pH =

15. pH = 7.024; $(H^+) =$

16. pH = 12.285; $(H^+) =$

17. pH = 8.628; $(H^+) =$

In the following formulas, substitute the given data and evaluate.

18. $N = 10 \log \left(\dfrac{P_1}{P_2}\right)$
 a. $P_1 = 250$, $P_2 = 14.5$; b. $P_1 = 8500$, $P_2 = 175$

19. $U = 1.15 \log \left(\dfrac{1 + r}{1 - r}\right)$
 a. $r = 0.480$; b. $r = 0.235$

20. $R = 342 \log (W_f - D)$
 a. $W_f = 57000$, $D = 2600$; b. $W_f = 126{,}000$, $D = 5500$

21. $H = \dfrac{1.44 \times 10^{-6}}{d^2 - \log \left(\dfrac{h}{d}\right)}$
 a. $d = 2.88$, $h = 12.6$; b. $d = 5.75$, $h = 260$

22. $N = n \times 10^k$
 a. $n = 6.42$, $k = 5.44$; b. $n = 2.63$, $k = -8.125$

23. $I = \dfrac{k \log(L/g) - (L/r)}{r^2}$
 a. $k = 470$, $L = 124$, $g = 32.2$, $r = 0.725$
 b. $k = 650$, $L = 165$, $g = 32.2$, $r = 2.33$

24. $Q = \sqrt{1 - \dfrac{\log N_1}{\log N_2}}$
 a. $N_1 = 340$, $N_2 = 5630$; b. $N_1 = .0715$, $N_2 = .00306$

25. $s = \dfrac{k - \pi \log \sin \Theta}{t}$
 a. $k = 4.06$, $\Theta = 51°$, $t = 1.77$
 b. $k = 2.75$, $\Theta = 156°$, $t = 3.52$

26. $\tan \dfrac{1}{2}\Theta = k \log \left(\dfrac{m}{n}\right)$; $(0° < \Theta < 180°)$
 a. $k = 1.064$, $m = 1250$, $n = 325$. Find Θ.
 b. $k = 1.133$, $m = 46.7$, $n = 0.314$. Find Θ.

Chapter 19

THE LOG LOG SCALES (LL); POWERS OF E

19.1 The LL1, LL2, and LL3 scales

It will be observed that these scales form a continuous scale starting with 1.01 at the left end of LL1, and ending at about 22,000 at the right end of LL3. Note that, unlike the C and D scales, the decimal point is indicated for all numbers on the LL scales.

The range of each scale is associated with powers of e as follows:

LL1 extends from $e^{.01}$ to $e^{.10}$ (1.01 to 1.105)
LL2 extends from $e^{.10}$ to $e^{1.0}$ (1.105 to 2.718)
LL3 extends from $e^{1.0}$ to e^{10} (2.718 to 22,026)

On the K & E Deci-Lon slide rule, these scales are labeled Ln1, Ln2, and Ln3. Also, on certain Pickett models, the LL scales (or N scales) are associated with powers of 10 rather than with powers of e; these scales are discussed in Appendix D.

19.2 Finding e^x when x is positive

The LL scales are related to the D scale in the following manner:

> If the hairline is moved over a number x on the **D** scale, then e^x is located under the hairline on the appropriate **LL** scale:
>
> For x between .01 and 0.1, e^x is on LL1
> For x between 0.1 and 1.0, e^x is on LL2
> For x between 1.0 and 10, e^x is on LL3
>
> The range of x associated with each LL scale is usually indicated on the body of the rule.

Example 1: $e^{3.5} = ?$

1. Move HL over 35 on D. Exponent is between 1 and 10; hence, answer will be on LL3.
2. Under HL read ***33.1*** on LL3.

Observe that:

$e^{0.35} = 1.419$ is also under HL on LL2.
$e^{.035} = 1.0356$ is under HL on LL1.

Example 2: $e^{0.57} = ?$

1. Move HL over 57 on D. Exponent is between 0.1 and 1.0; hence, answer will be read on LL2.
2. Under HL read ***1.768*** on LL2.

Example 3: $e^{.042} = ?$

1. Move HL over 42 on D. Exponent is between .01 and 0.1; hence, answer is on LL1.
2. Under HL read ***1.0429*** on LL1.

Verify the following:

1. $e^{0.64} = 1.897$; $e^{.064} = 1.0661$
2. $e^{.0155} = 1.01561$; $e^{1.55} = 4.71$
3. $e^{3.55} = 34.8$; $e^{.0355} = 1.0361$
4. $e^{9.44} = 12{,}500$
5. $e^{0.835} = 2.305$
6. $e^{.0256} = 1.0259$

Example 4: $125\, e^{1.85} = ?$

1. First evaluate $e^{1.85} = 6.36$. Now transfer this to the D scale and multiply by 125:
2. Verify that $125\, e^{1.85} = 125 \times 6.36 =$ ***795.***

Exercise 19-1

1. $e^{1.55} =$
2. $e^{0.155} =$
3. $e^{.0155} =$
4. $e^{2.9} =$
5. $e^{.029} =$
6. $e^{5.6} =$
7. $e^{0.83} =$
8. $e^{0.28} =$
9. $e^{.0425} =$
10. $e^{.088} =$

11. $e^{1.075} =$
12. $e^{0.63} =$
13. $e^{2.05} =$
14. $e^{0.115} =$
15. $e^{.026} =$
16. $e^{7.22} =$
17. $e^{0.37} =$
18. $e^{9.4} =$
19. $e^{.0128} =$
20. $e^{1.245} =$
21. $e^{4.66} =$
22. $75\ e^{0.805} =$
23. $4300\ e^{.0177} =$
24. $.064\ e^{4.16} =$
25. $42.6\ e^{0.104} =$
26. $e^{.064} \sin 38° =$
27. $e^{0.52} \sin(0.24\pi) =$
28. $e^{3.06} \cos 73.5° =$
29. $e^{6.4} \cos 87° =$
30. $e^{.0227} \tan 1.25 =$

19.3 The "A-related" LL0 and LL00 scales

Some slide rules have the following described log log scales which are associated with the A scale:

1. A scale labeled "LL0" starting at 0.999 at the left and decreasing down to 0.905 on the right.
2. A scale labeled "LL00" which is a continuation of "LL0," and starts at 0.905 on the left, decreasing down to .000045 on the right.

The use of these scales is discussed in Appendix C.

19.4 The LLo1, LLo2, and LLo3 scales

These scales form a continuous scale starting at 0.990 at the left end of LLo1, and decreasing down to .000045 at the right end of LLo3. Thus, the scales are read from right to left in the same manner as we read the CI scale.

The range of each scale is associated with negative powers of e as follows:

LLo1 extends from $e^{-.01}$ to $e^{-.10}$ (.990 to .905)
LLo2 extends from $e^{-.10}$ to $e^{-1.0}$ (.905 to .368)
LLo3 extends from $e^{-1.0}$ to e^{-10} (.368 to .000045)

On the K & E Deci-Lon rule, these are labeled Ln-1, Ln-2, and Ln-3; on the Post Versalog model they are labeled LL/1, LL/2, and LL/3. The Pickett rules have these scales "back to back" with the LL1, LL2, and LL3 scales. Thus, the LL1 scale and the scale corresponding to LLo1 both share the same scale axis; the LL1 markings are above the axis (reading from left to right), and the LLo1 markings are below (reading from right to left).

19.5 Finding e^x when x is negative

The scales described in the preceding section are related to the D scale in the same manner as are the other LL scales. Hence, we may raise e to a negative power by reading directly from D to the appropriate LL scale, bearing in mind the range associated with each scale. Again, these ranges are usually indicated on the body of the rule.

Example 1: $e^{-.056} = ?$

1. Move HL over 56 on D. Exponent is between $-.01$ and -1.0; hence, answer is on LLo1.
2. Under HL read ***0.9455*** on LLo1.

Example 2: $e^{-4.51} = ?$

1. Move HL over 451 on D. Exponent is between -1.0 and -10; hence, answer is found on LLo3.
2. Under HL read ***.0110*** on LLo3.

Verify the following:

1. $e^{-2.65} = .0707$; $e^{-0.265} = 0.767$
2. $e^{-.064} = 0.938$; $e^{-6.4} = .00166$
3. $e^{-1.47} = 0.230$; $e^{-.0147} = 0.9854$
4. $e^{-.025} = 0.9753$
5. $170\ e^{-3.6} = 4.64$
6. $e^{-1.6} \sin 0.32 = .0635$

Exercise 19-2

1. $e^{-2.65} =$
2. $e^{-0.265} =$
3. $e^{-.064} =$
4. $e^{-6.4} =$
5. $e^{-3.7} =$
6. $e^{-1.03} =$
7. $e^{-.035} =$
8. $e^{-5.76} =$
9. $e^{-.073} =$
10. $e^{-0.1045} =$
11. $e^{-.0148} =$
12. $e^{-9.4} =$
13. $e^{-4.85} =$
14. $e^{-0.76} =$
15. $e^{-.066} =$
16. $e^{-0.10} =$
17. $e^{-3.22} =$
18. $e^{-0.28} =$

19. $e^{-0.1175} =$

20. $e^{-1.36} =$

21. $4.3\ e^{-0.14} =$

22. $12.6\ e^{-0.44} =$

23. $2700\ e^{-7.55} =$

24. $545\ e^{-.0147} =$

25. $68.2\ e^{-1.53} =$

26. $\sqrt{475}\ e^{-8.66} =$

27. $e^{-0.66} \sin 69° =$

28. $e^{-1.23} \sin(0.22\,\pi) =$

29. $e^{-.054} \cos 1.3 =$

30. $e^{-3.72} \tan 87.2° =$

19.6 An approximation for e^x (x near zero)

When raising e to very small powers, the following approximate relations are useful:

> If x is a positive number near zero:
> 1. $e^x \approx 1 + x$
> 2. $e^{-x} \approx 1 - x$

Example 1: $e^{.0042} = ?$

This is outside the range of LL1; hence, we use relation (1).

$$e^{.0042} \approx 1 + .0042 = \mathbf{1.0042} \text{ (approx.)}$$

Example 2: $e^{-.00074} = ?$

This is outside the range of LLo1; hence, we use relation (2).

$$e^{-.00074} \approx 1 - .00074 = \mathbf{0.99926} \text{ (approx.)}$$

19.7 Evaluating e^x for large positive or negative values of x

Example 1: $e^{11.6} = ?$

This is beyond the range of LL3; hence, we write:

$$e^{11.6} = (e^{5.8})^2$$

We may now find $e^{5.8}$ and square the result.
Verify that $e^{11.6} = (e^{5.8})^2 = (330)^2 =$ ***108,900.***

Example 2: $e^{-22} = ?$

This is beyond the range of LLo3; hence, we write:

$$e^{-22} = (e^{-22/3})^3 = (e^{-7.33})^3$$

Verify that $e^{-22} = (.00065)^3 = \mathbf{2.74 \times 10^{-10}}$.

19.8 Slide rules with 8 LL scales

In the preceding sections we have described the six basic LL scales found on most conventional slide rules. However, some rules feature two additional scales in the neighborhood of 1. One of these extends from $e^{.001}$ to $e^{.01}$ (1.001 to 1.01), and we shall refer to this as the LL0 scale (on the Deci-Lon rule it is labeled Ln0). The other scale extends from $e^{-.001}$ to $e^{-.01}$ (0.999 to 0.990), and we shall refer to this as the LLo0 scale (on the Versalog rule this scale is labeled LL/0; on the Deci-Lon rule it is labeled Ln-0). If your slide rule has these scales, you may check the following examples:

Example 1: $e^{.0055} = ?$

1. Move HL over 55 on D.
2. Under HL read ***1.00551*** on LL0 (Ln0).

Example 2: $e^{-.0027} = ?$

1. Move HL over 27 on D.
2. Under HL read ***0.99731*** on LLo0 (Ln-0, LL/0).

Verify the following (use LL0 and LLo0 scales):

1. $e^{.009} = 1.00904$
2. $e^{.00235} = 1.00235$
3. $e^{.00421} = 1.00422$
4. $e^{-.006} = 0.99402$
5. $e^{-.00485} = 0.99517$
6. $e^{-.00163} = 0.99837$

If you check the foregoing examples using the approximate formulas of section 19.6, you will discover very close agreement.

Exercise 19-3

1. $e^{.0034} =$
2. $e^{-.00061} =$
3. $e^{.0072} =$
4. $e^{-.0014} =$

5. $e^{-.00037} =$

6. $e^{.000021} =$

7. $e^{13} =$

8. $e^{24} =$

9. $12{,}500\ e^{-15} =$

10. $475{,}000\ e^{-12} =$

19.9 Exponent in combined form

Example 1: $e^{(.032 \times 16.5)} = ?$

By approximation, the exponent is about 0.5; hence, result will be on LL2.

1. Set left index of C opposite 165 on D.
2. Move HL over 32 on C. HL is now over the product (.032 × 16.5) on D; therefore, *e* raised to this power is under HL on LL2.
3. Under HL read ***1.695*** on LL2.

Example 2: $e^{-35/16} = ?$

Exponent is about −2; hence, result will be on LLo3.

1. Move HL over 35 on D.
2. Slide 16 on C under HL.
3. Move HL over left index of C.
4. Under HL read ***0.112*** on LLo3.

Verify the following:

1. $e^{(2.3 \times 1.7)} = 50.0$
2. $e^{-2/7} = 0.7515$
3. $e^{26/\pi} = 3900$
4. $e^{1/15} = 1.0690$
5. $e^{(-.018 \times 4.7)} = 0.9190$
6. $e^{(-.0072 \times 16.2)} = 0.8900$

Example 3: Evaluate $e^{.016t}$ when (a) $t = 2.5$, (b) $t = 35$, (c) $t = 425$. By approximation, we see that the result of (a) will be on LL1, the result of (b) will be on LL2, and the result of (c) will be on LL3.

1. Set left index of C opposite 16 on D.
2. Move HL over 25 on C. Under HL read ***1.0408*** on LL1.
3. Move HL over 35 on C. Under HL read ***1.751*** on LL2.
4. Move HL over 425 on C. Under HL read ***900*** on LL3.

Exercise 19-4

1. $e^{3.2 \times 1.6} =$
2. $e^{1/9} =$
3. $e^{0.64 \times 5.2} =$
4. $e^{-.015 \times 2.7} =$
5. $e^{-3/7} =$
6. $e^{.06/2.4} =$
7. $e^{1/27} =$
8. $e^{.0045 \times 15} =$
9. $\sqrt[4]{e} =$
10. $e^{-1/6} =$
11. $e^{-11/240} =$
12. $e^{.06/4.2} =$
13. $e^{(1.3 \times 6.4)/15} =$
14. $e^{-(2.4 \times 6.1)/5} =$
15. $e^{(.0034 \times 2.6)/.015} =$
16. $e^{\pi/4} =$
17. $1600\, e^{-3\pi/2} =$
18. $e^{-(.035 \times 165)/77} =$
19. $e^{1/(2.6 \times 6.3)} =$
20. $e^{-5/(2.7 \times 0.43)} =$
21. $2.75\, e^{1/5.3} =$
22. $1250\, e^{-1.1\pi} =$
23. $e^{-7/60} \sin\left(2 - \frac{1}{2}\pi\right) =$
24. $e^{.024 \times 130} \sin(\pi/60) =$
25. $e^{-0.33 \times 11.4} \tan 72.3° =$
26. Evaluate $e^{-.0023t}$ for $t = 15, 30, 1250$.
27. Evaluate $e^{.073t}$ for $t = 15, 55, 110$.
28. Evaluate $e^{650/x}$ for $x = 125, 550, 12{,}000$.
29. Evaluate $450\, e^{-.063t}$ for $t = 25, 85, 125$.
30. Evaluate $1250\, e^{65/x}$ for $x = 12, 175, 2300$.

19.10 Hyperbolic functions

Certain combinations of exponential functions occur frequently in applied mathematics, and are known as "hyperbolic functions." They are defined as follows:

1. $\sinh x = \frac{1}{2}(e^x - e^{-x})$ (read "hyperbolic sine of x")
2. $\cosh x = \frac{1}{2}(e^x + e^{-x})$ (read "hyperbolic cosine of x")
3. $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x} - 1}{e^{2x} + 1}$ (read "hyperbolic tangent of x")

Definitions are also given for $\coth x$, $\operatorname{sech} x$, and $\operatorname{csch} x$; however, the listed functions are encountered most frequently.

Example 1: $\sinh 0.36 = ?$

1. From the definition: $\sinh 0.36 = \frac{1}{2}(e^{0.36} - e^{-0.36})$.

2. Verify that $\sinh 0.36 = \frac{1}{2}(1.434 - 0.698) = \mathbf{0.368}$.

Example 2: $\cosh \pi/2 = ?$

1. From the definition: $\cosh \pi/2 = \frac{1}{2}(e^{\pi/2} + e^{-\pi/2})$.

2. Verify that $\cosh \pi/2 = \frac{1}{2}(4.81 + 0.208) = \mathbf{2.51}$.

Verify the following:

1. $\sinh 0.63 = 0.673$

2. $\cosh 1.35 = 2.06$

3. $\cosh(-0.84) = 1.374$

4. $\tanh 0.18 = 0.1781$

5. $\sinh(-\pi/7) = -0.464$

6. $\tanh(-1.10) = -0.800$

19.11 The Sh and Th scales

Certain slide rules have scales marked Sh and Th which enable you to read $\sinh x$ and $\tanh x$ directly. These scales are discussed in Appendix E.

Exercise 19-5

1. $\sinh 0.48 =$

2. $\cosh 1.25 =$

3. $\cosh -0.750 =$

4. $\tanh 1.43 =$

5. $\sinh -0.26 =$

6. $\cosh 0.09 =$

7. $\tanh -0.65 =$

8. $\sinh \pi/4 =$

9. $\cosh \pi/6 =$

10. $\tanh(.036 \times 5.7) =$

11. $\sinh \sqrt{1.75} =$

12. $\sinh(75/120) =$

13. A flexible cable hangs in a curve whose equation is: $y = 8.75 \cosh(x/8.75)$. Find y when x takes the values: 0, 5, 10, and 20.

14. If air resistance is taken into account, the velocity of a falling object is given by the formula: $V = P \tanh(2gt/P)$, where $P = \sqrt{W/k}$.

a. If $W = 2000$ lbs, $k = .0037$, $g = 32.2$ ft per sec^2, calculate V for $t = 0$ and $t = 15$ sec.

b. What value does V approach as t gets larger and larger?

19.12 Finding logarithms to base *e*

You will recall that if $N = e^x$, then $x = \log_e N$. We refer to $\log_e N$ as the *natural log* of N, and we shall write it *ln N.*

With reference to $N = e^x$, it is clear, then, that finding ln N amounts to determining the value of the exponent x corresponding to a known N. This is just the inverse of finding e^x. Accordingly, the relation between the LL scales and the D scale may be restated as follows:

> If the hairline is moved over a number N on the **LL** scale, then *ln N* will be under the hairline on the **D** scale.
>
> Conversely, if the hairline is over *ln N* on the **D** scale, then N will be under the hairline on the appropriate **LL** scale.

If your slide rule has a legend on the body indicating the range of the exponent associated with each LL scale, then these ranges, of course, also apply to ln N. In the following examples, you will find it helpful to refer to these range indications.

Example 1: ln 4.5 = ?

1. Move HL over 4.5 on LL3.
2. Under HL read "1504" on D. N is on LL3; hence, answer must be between 1 and 10. Therefore, ln 4.5 = ***1.504.***

Example 2: ln 1.0445 = ?

1. Move HL over 1.0445 on LL1.
2. Under HL read "435" on D. N is on LL1; hence, ln N is between .01 and 0.1. Answer is ***.0435.***

Example 3: ln 0.724 = ?

1. Move HL over 0.724 on LLo2.
2. Under HL read "323" on D. N is on LLo2; therefore, ln N is between −0.1 and −1.0. Answer is ***−0.323.***

Verify the following:

1. ln 68 = 4.22

2. ln 1.0223 = .0220

3. ln 0.814 = −0.206

4. ln 1.545 = 0.435

5. ln .0265 = −3.63

6. ln 0.9436 = −.0581

Example 4: ln N = 2.66; N = ?

We know ln N and we must find N; hence, we read from D to the proper LL scale ($N = e^{2.66}$).

1. Move HL over 266 on D. Exponent (or ln N) is between 1 and 10; hence, N will be located on LL3.
2. Under HL read ***14.3*** on LL3.

Example 5: ln N = −0.662; N = ?

1. Move HL over 662 on D. Exponent (or ln N) is between −0.1 and −1.0; hence, N will be located on LLo2.
2. Under HL read ***0.516*** on LLo2.

Verify the following:

1. Given ln N = 5.1; verify that N = 164.

2. Given ln N = −.0345; verify that N = 0.9661.

3. Given ln N = 0.720; verify that N = 2.055.

4. Given ln N = −1.84; verify that N = 0.158.

Exercise 19-6

1. ln 375 =

2. ln 1.23 =

3. ln 0.175 =

4. ln 2.85 =

5. ln 0.9075 =

6. ln 8.45 =

7. ln .00066 =

8. ln 164 =

9. ln 1.01645 =

10. ln 2500 =

11. ln .0034 =

12. ln 36.5 =

13. ln 1.01245 =

14. ln 16,500 =

15. ln 0.382 =

16. ln 2.77 =

17. ln .000175 =

18. ln 1.0103 =

19. ln 0.377 =

20. ln 8400 =

21. ln 15 =

22. ln .066 =

23. ln 210 =

24. ln N = 1.84; N =

25. ln N = 0.63; N =

26. ln N = −4.75; N =

27. ln N = −.082; N =

28. ln N = 7.2; N =

29. ln N = .0274; N =

30. ln N = −0.46; N =

19.13 Combined operations with ln *N*

Example 1: $4.5 \ln 2 = ?$

1. Move HL over 2 on LL2. This puts HL over ln 2 on D (note that its value is about 0.7). Now to multiply by 4.5:
2. Slide right index of C under HL. Move HL over 45 on C.
3. Under HL read "312" on D. Answer is ***3.12.***

Clearly, we could have divided by the reciprocal of 4.5 instead of multiplying as we did.

Example 2: $\frac{\ln 0.063}{7} = ?$

1. Move HL over .063 on LLo3. HL is now over ln 0.063 on D (note that its value is about −3). Now divide by 7:
2. Slide 7 on C under HL.
3. Opposite right index of C read "395" on D. Answer is ***−0.395.***

Observe that, because ln N appears on the D scale, it can easily be multiplied or divided by other numbers. This is an important feature of the LL scales, and in the next chapter we shall see how it enables us to evaluate powers of numbers in general.

Exercise 19-7

1. 1.4 ln 4.35 =

2. $\frac{\ln 7.44}{5.4} =$

3. $14.6 \ln 1.0164 =$

4. $\dfrac{\ln 0.814}{3.26} =$

5. $3.74 \ln 0.725 =$

6. $2.6 \ln 3.44 =$

7. $4.7 \ln 0.9645 =$

8. $0.38 \ln 650 =$

9. $\dfrac{\ln 1.84}{6.22} =$

10. $\dfrac{\ln 1650}{24.2} =$

11. $\dfrac{\ln .0145}{4.6} =$

12. $\dfrac{\ln .0044}{.0745} =$

13. $\dfrac{2.4 \ln 1.0246}{1.7} =$

14. $\dfrac{\pi \ln 115}{45} =$

15. $\dfrac{2.6 \ln 0.934}{3.55} =$

16. $\dfrac{6.8 \ln 0.564}{2.7} =$

Chapter 20

POWERS OF ANY POSITIVE NUMBER

20.1 Finding b^x (x between 1 and 10)

In this section we consider the evaluation of b^x where the base, b, may be any positive number, and the exponent, x, takes values between 1 and 10.

To illustrate the procedure, suppose you now go through the following moves on your slide rule:

1. Move HL over 3 on the LL3 scale.
2. Slide left index of C under HL.
3. Move HL over 2 on C. HL is now over 9 on LL3. Note that 9 is 3^2.
4. Move HL over 3 on C. HL is now over 27 on LL3. Note that 27 is 3^3.
5. Move HL over 4 on C. HL is now over 81 on LL3. Note that 81 is 3^4.

These observations suggest that if we set the C index opposite the base, b, on the LL scale, and then move the hairline over the exponent, x, on the C scale, we will find the power, b^x under the hairline on the LL scale. This is similar to multiplication, except that instead of using C and D, we use C and LL.

To see what actually is happening, consider the following examples:

Example 1: $4^3 = ?$

Suppose we let $N = 4^3$, and take natural log of both sides:

$$\ln N = \ln 4^3 = 3 \ln N$$

Now evaluate 3 ln 4:

1. Move HL over 4 on LL3. Note that HL is now over ln 4 on D; hence, it is easy to multiply by 3.

2. Slide left index of C under HL.
3. Move HL over 3 on C. HL is now over ln N on D, which means that it is over N on LL.
4. Under HL read 64 on LL3.

To evaluate $N = b^x$, then, we first move the hairline over the base b on the LL scale; this puts the hairline over ln b on the D scale. We then multiply ln b by the exponent x; this locates ln N on D, and N itself on the appropriate LL scale (see Figure 20.1).

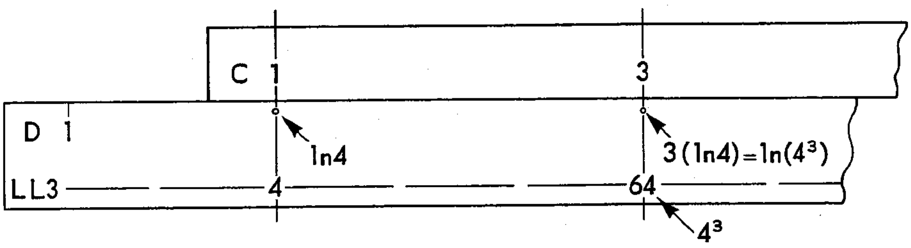

Figure 20.1

As stated in the previous chapter, the powerful feature of the LL scales is that ln b appears on the D scale, which is an operational scale, thus making it easy to further multiply or divide the logarithm by other numbers.

Example 2: Evaluate $(1.3)^x$ for $x = 2, 3, 4.5$, and 7.6.

1. Move HL over 1.3 on LL2. This puts HL over ln 1.3 on D. Now multiply the logarithm successively by the given values of x; the corresponding powers will be read on the LL scale.
2. Slide left index of C under HL.
3. Move HL over 2 on C. Under HL read ***1.69*** on LL2.
4. Move HL over 3 on C. Under HL read ***2.197*** on LL2.
5. Now we see that 4.5 on C is somewhat beyond the LL2 scale; hence answer must be on LL3 (which is just a continuation of LL2). Therefore, we interchange indexes and move HL over 45 on C. Under HL read ***3.26*** on LL3.
6. Move HL over 76 on C. Under HL read ***7.35*** on LL3.

In the foregoing example, you will notice that when the answer was to the *right* of the base, 1.3, it was located on the *same scale* (LL2). After interchanging indexes, the answer was to the *left* of 1.3, and it was located *one scale higher* on LL3.

Example 3: Evaluate $(0.70)^2$ and $(0.70)^{4.3}$.

1. Move HL over 0.70 on LLo2. This puts HL over ln 0.70 on D. Now multiply by 2, and then by 4.3.
2. Slide left index of C under HL.
3. Move HL over 2 on C. Under HL read ***0.49*** on LLo2.
4. Interchange indexes and move HL over 43 on C. Under HL read ***0.216*** on LLo3.

Here again, you will notice that when the answer was to the *right* of the base, 0.70, it was located on the *same scale* (LLo2). When the answer was to the *left* of 0.70, it was located *one scale higher* on LLo3. It should be understood that by a higher scale, we mean a scale with a higher number designation; for example, LL2 is "higher" than LL1, LLo3, is "higher" than LLo2, and so forth.

The procedure, then, may be summarized:

> *To evaluate b^x (x between 1 and 10):*
> 1. Move HL over b on LL scale. HL is now over ln b on D.
> 2. Multiply by x (slide C index under HL, move HL over x on C).
> 3. Under HL read b^x on the appropriate LL scale:
>   a. If the answer is to the *right* of b, it is on the *same* LL scale as b.
>   b. If the answer is to the *left* of b, it is located *one scale higher* than b.

Clearly, in step (2), we may multiply by x in the manner described, or we may divide by the reciprocal of x using the CI scale.

Example 4: $(14.2)^{3.3} = ?$

1. Move HL over 14.2 on LL3.
2. Slide left index of C under HL.
3. Move HL over 33 on C. Note that HL is to the *right* of 14.2; hence, answer is on the *same scale* (LL3).
4. Under HL read ***6400*** on LL3.

Example 5: $(1.055)^{6.25} = ?$

1. Move HL over 1.055 on LL1.
2. Slide right index of C under HL.
3. Move HL over 625 on C. HL is to the *left* of 1.055; hence, answer is *one scale higher* on LL2.
4. Under HL read ***1.398*** on LL2.

Example 6: $(.9766)^{3.4} = ?$

1. Move HL over .9766 on LLo1.
2. Slide left index of C under HL.
3. Move HL over 34 on C. HL is to the *right* of .9766; hence, answer is on the *same scale* (LLo1).
4. Under HL read ***0.9226*** on LLo1.

Example 7: $(0.864)^{9.2} = ?$

1. Move HL over 0.864 on LLo2. Note that slide will be in better position if we divide by the reciprocal of 9.2:
2. Slide 92 on CI under HL.
3. Move HL over left index of C. Observe that HL is to the *left* of 0.864; hence, answer is *one scale higher* on LLo3.
4. Under HL read ***0.261*** on LLo3.

Example 8: $\left(\frac{585}{126}\right)^{3.3} = ?$

1. Dividing on C-D, we find 585/126 = 4.64.
2. Verify that $(4.64)^{3.3} =$ ***158.***

Exercise 20-1

1. $(4)^{2.7} =$
2. $(8)^{1.35} =$
3. $(11)^{3.4} =$
4. $(1.25)^{1.8} =$
5. $(1.6)^{7} =$
6. $(1.044)^{5} =$
7. $(0.21)^{4} =$
8. $(0.56)^{6} =$
9. $(0.98)^{3.2} =$
10. $(0.94)^{4.5} =$
11. $(4.5)^{4} =$
12. $(7.3)^{3} =$
13. $(1.26)^{1.7} =$
14. $(0.75)^{3} =$
15. $(0.625)^{5} =$
16. $(1.1645)^{3.2} =$
17. Evaluate $(1.364)^x$ for $x = 2.2, 4.5, 7.3$.
18. Evaluate $(1.046)^x$ for $x = 1.8, 3.7, 10$.
19. Evaluate $(0.787)^x$ for $x = 4.5, 7.9, 10$.
20. Evaluate $(0.9515)^x$ for $x = 1.65, 2.80, 6.45$.
21. $(1.0534)^{7.5} =$
22. $(1.226)^{3.4} =$
23. $(5.75)^{4.6} =$
24. $(1.0945)^{1.1} =$
25. $(0.186)^{4.9} =$
26. $(1.534)^{5.75} =$
27. $(35.5)^{2.75} =$
28. $(1.0136)^{5.26} =$
29. $(0.8815)^{8.4} =$
30. $(.057)^{3.3} =$

31. $(0.954)^{3.6} =$

32. $(1.228)^{9.3} =$

33. $(0.354)^{6.72} =$

34. $(1.133)^{8.5} =$

35. $(1.765)^{3.45} =$

36. $(0.107)^{4.15} =$

37. $(0.444)^{1.15} =$

38. $(0.9445)^{7.64} =$

39. $(12.5)^{2.31} =$

40. $(0.9824)^{2.66} =$

41. $(0.634)^{5.75} =$

42. $(0.388)^{6.44} =$

43. $(2.84)^{6.9} =$

44. $(1.0366)^{8.4} =$

45. $(0.9863)^{7.6} =$

46. $(1.0545)^{1.93} =$

47. $\left(\frac{68}{85}\right)^{3.6} =$

48. $\left(\frac{244}{107}\right)^{2.8} =$

49. $\left(\frac{373}{950}\right)^{4.4} =$

50. $\left(\frac{0.37}{0.17}\right)^{6.8} =$

20.2 The "scale-shift" principle

The LL scales have an important property which we shall refer to as the "scale-shift" principle:

> If the HL is over b^x on one of the LL scales, then:
> 1. b^{10x} will be under HL on the next *higher* scale.
> 2. $b^{x/10}$ will be under HL on the next *lower* scale.

In other words, the decimal point in the exponent shifts *one place to the right* each time we move *one scale higher;* it shifts *one place to the left* each time we move *one scale lower.*

Example 1: **a.** $(1.2)^2 = ?$ **b.** $(1.2)^{20} = ?$ **c.** $(1.2)^{0.2} = ?$

a. Verify that $(1.2)^2$ is under HL on LL2 and is equal to ***1.44.***

b. Now, leaving the hairline in the same position, the "scale-shift" principle tells us that $(1.2)^{20}$ is located one scale higher on LL3 (decimal point in exponent has been shifted one place to the right). Under HL read ***38.3*** on LL3.

c. Again applying the "scale-shift" principle, we find $(1.2)^{0.2}$ on LL1 (one scale lower than LL2). Under HL read ***1.0372.***

These readings are illustrated in Figure 20.2.

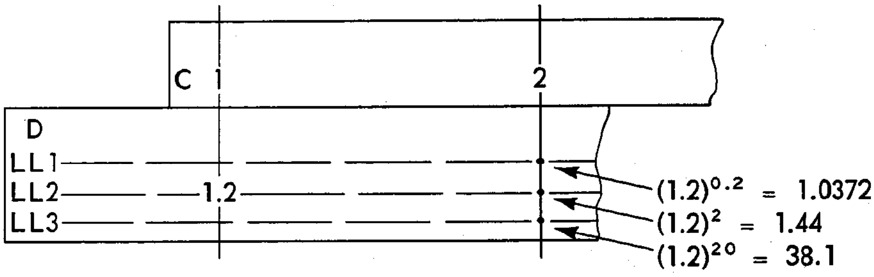

Figure 20.2

Example 2: **a.** $(.064)^2 = ?$ **b.** $(.064)^{.02} = ?$

a. Verify that $(.064)^2$ is under HL on LLo3, and is equal to ***.0041***.

b. Leaving hairline in the same position and applying the "scale-shift" principle, we locate $(.064)^{.02}$ two scales lower on LLo1 (decimal point in exponent has been shifted two places to the left). Under HL read ***0.9465*** on LLo1.

Example 3: $(2.1)^{.075} = ?$

a. First locate $(2.1)^{7.5}$. Verify that this is found under HL on LL3.

b. Applying "scale-shift" principle, $(2.1)^{.075}$ is located two scales lower on LL1. Verify that answer is ***1.0573.***

20.3 A general procedure for finding b^x (x positive)

The foregoing examples suggest how we may locate the answer when we are dealing with the more general case of b^x; that is, when the exponent is not restricted to a number between 1 and 10.

Example 1: $(1.015)^{300} = ?$

We first evaluate assuming that the exponent is between 1 and 10; we then apply the "scale-shift" principle to properly locate the answer.

1. Move HL over 1.015 on LL1. Slide left index of C under HL.
2. Move HL over 3 on C. Observe that HL is to the right of 1.015; hence, if exponent were 3.00 answer would be on the same scale (LL1). However, decimal point in exponent is actually *two* places to the *right* of this assumed position; therefore, answer is located *two* scales *higher* on LL3.
3. Under HL read ***87*** on LL3.

The procedure is illustrated in Figure 20.3.

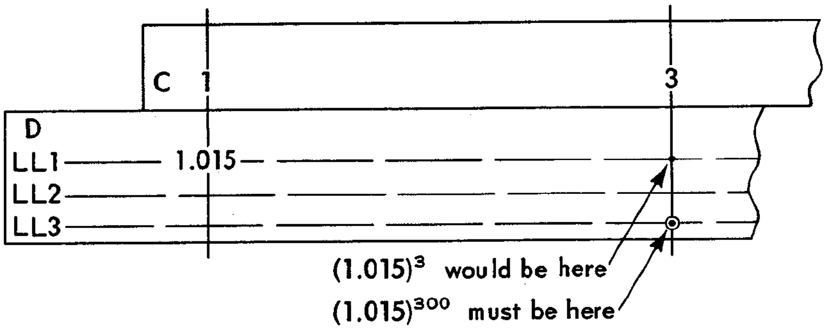

Figure 20.3

Example 2: $(1.0185)^{24} = ?$

1. Move HL over 1.0185 on LL1. Slide left index of C under HL.
2. Move HL over 24 on C. Observe that HL is to the right of 1.0185; hence, if exponent were 2.4, answer would be on the same scale (LL1). However, decimal point in exponent is actually *one* place to the *right* of this position; therefore, answer is located *one* scale *higher* on LL2.
3. Under HL read ***1.553*** on LL2.

Example 3: $(1.85)^{.07} = ?$

1. Move HL over 1.85 on LL2. Slide right index of C under HL.
2. Move HL over 7 on C. Note that HL is to the left of 1.85, hence if exponent were 7.00, answer would be on LL3. However, decimal point in exponent is actually *two* places to the *left* of this position; therefore, answer is located *two* scales *lower* on LL1.
3. Under HL read ***1.044*** on LL1.

We illustrate in Figure 20.4

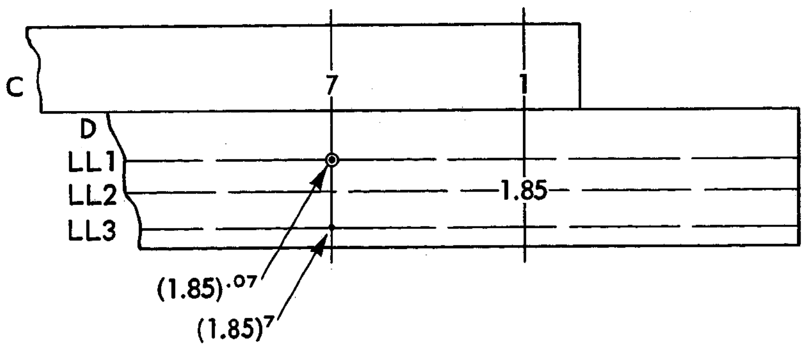

Figure 20.4

Example 4: $(62)^{.0073} = ?$

1. Move HL over 62 on LL3. Slide right index of C under HL.
2. Move HL over 73 on C. Note that HL is to the left of 62; hence, if exponent were 7.3, answer would be on "LL4" (if our slide rule had such a scale). However, decimal point in exponent is three places to the left of this assumed position; therefore, answer is located three scales lower on LL1.
3. Under HL read ***1.0306*** on LL1.

Example 5: $(0.82)^{0.175} = ?$

1. First locate $(0.82)^{1.75}$ Verify that this is on LLo2.
2. Applying "scale-shift" principle, $(0.82)^{0.175}$ is located one scale lower on LLo1. Verify that answer is ***0.9658.***

Example 6: $(0.964)^{25} = ?$

1. Verify that $(0.964)^{2.5}$ is on LLo1.
2. Applying "scale-shift" principle, $(0.964)^{25}$ will be located one scale higher on LLo2. Verify that answer is ***0.40.***

Example 7: $(.00325)^{.071} = ?$

1. Move HL over .00325 on LLo3. Slide right index of C under HL.
2. Move HL over 71 on C. HL is to the left of .00325; hence, if exponent were 7.1, answer would be on "LLo4" (if such a scale were present). Decimal point in exponent is actually two places to the left of this position; therefore, answer is two scales lower on LLo2.
3. Under HL read ***0.666*** on LLo2.

The procedure illustrated in the preceding examples may be formally stated:

> *To locate the result when evaluating b^x (x positive):*
> 1. Determine the scale on which result would be found if decimal point in exponent were shifted to make it a number between 1 and 10.
> 2. Applying the "scale-shift" principle, move up or down as many scales as are necessary to adjust the decimal point to its true position.

Verify the following:

1. $(1.25)^{21} = 108$
2. $(28.5)^{.025} = 1.0874$

3. $(4700)^{.0032} = 1.0274$

4. $(0.9474)^{46} = .0835$

5. $(0.48)^{0.67} = 0.612$

6. $(1.01865)^{400} = 1610$

7. $(.0145)^{.0029} = 0.9878$

8. $(0.223)^{.083} = 0.883$

Exercise 20-2

1. $(1.186)^{34} =$
2. $(37)^{0.22} =$
3. $(145)^{.063} =$
4. $(1.164)^{3.7} =$
5. $(0.66)^{0.321} =$
6. $(0.59)^{.05} =$
7. $(1.244)^{25} =$
8. $(1.0665)^{7.3} =$
9. $(0.925)^{0.64} =$
10. $(0.50)^{0.265} =$
11. $(36)^{.08} =$
12. $(7500)^{.04} =$
13. $(1.174)^{0.55} =$
14. $(1.63)^{11.2} =$
15. $(1.107)^{45} =$
16. $(0.513)^{.035} =$
17. $(.00015)^{0.57} =$
18. $(1.024)^{100} =$
19. $(0.884)^{.092} =$
20. $(1.0625)^{0.74} =$
21. $(10)^{0.485} =$
22. $(100)^{.0064} =$
23. $(20{,}000)^{.0018} =$
24. $(0.9425)^{22.4} =$
25. $(1.425)^{.06} =$
26. $(0.568)^{9.5} =$
27. $(10)^{.041} =$
28. $(.9814)^{410} =$
29. $(0.117)^{0.255} =$
30. $(0.908)^{42} =$
31. $(1.264)^{31} =$
32. $(0.642)^{7.3} =$
33. $(650)^{.056} =$
34. $(.0245)^{.004} =$
35. $(1.375)^{18} =$
36. $(5200)^{.0021} =$
37. $(.00064)^{0.45} =$
38. $(1.1025)^{0.33} =$
39. $(1.214)^{47} =$
40. $(18{,}000)^{.027} =$
41. $(0.9684)^{290} =$
42. $(.00075)^{.062} =$
43. $(1.076)^{125} =$
44. $400(1 + .045)^{20} =$

45. $250(1 + .0375)^{45} =$

46. $24(1 + .03)^{300} =$

47. $\left(\frac{23}{72}\right)^{.036} =$

48. $\left(\frac{760}{635}\right)^{23} =$

49. $\left(\frac{5.34}{18.6}\right)^{0.55} =$

50. $\left(\frac{1.450}{.0026}\right)^{.075} =$

20.4 The reciprocal property of the LL scales

We have seen that the LL3 scale ranges from $e^{1.0}$ to e^{10}, whereas the LLo3 scale ranges from $e^{-1.0}$ to e^{-10}. This means that opposite readings on these two scales are reciprocals of each other. Similarly, the LL2 scale extends from $e^{0.1}$ to $e^{1.0}$, whereas the LLo2 scale extends from $e^{-0.1}$ to $e^{-1.0}$, and so forth. As a result, we may describe the following useful property of the LL scales:

> If the hairline is set over a number on LL3, its reciprocal will be under the hairline on LLo3, and vice versa.
>
> A similar reciprocal relation holds between LL2 and LLo2, and between LL1 and LLo1.

Example 1: Use LL scales to find reciprocal of 36.5.

1. Move HL over 36.5 on LL3.
2. Under HL read ***.0275*** on LLo3.

Example 2: 1/.0735 = ?

1. Move HL over .0735 on LLo3.
2. Under HL read ***13.6*** on LL3.

Example 3: $(1.1635)^{-1} = ?$

1. Move HL over 1.1635 on LL2.
2. Under HL read ***0.8595*** on LLo2.

Verify the following:

1. 1/27 = .037

2. 1/350 = .00286

3. 1/0.645 = 1.55

4. 1/.00085 = 1180

5. $1/7.75 = 0.129$

6. $(.00235)^{-1} = 425$

7. $(14{,}500)^{-1} = .00007$

8. $(1.107)^{-1} = 0.9034$

9. $(0.383)^{-1} = 2.61$

10. $(1.0344)^{-1} = 0.9668$

20.5 Finding b^x (x negative)

Here, the reciprocal scales may be used to advantage as shown in the following examples:

Example 1: $5^{-4} = ?$

We simply proceed as if to evaluate 5^4, then read the reciprocal.

1. Move HL over 5 on LL3. Slide left index of C under HL.
2. Move HL over 4 on C. Now 5^4 is under HL on LL3; hence, 5^{-4} will be on LLo3.
3. Under HL read ***.00160*** on LLo3.

Example 2: $(0.15)^{-.25} = ?$

Verify that $(0.15)^{.25}$ is on LLo2; therefore, $(0.15)^{-.25}$ is found on LL2. Result is ***1.607.***

Example 3: $\left(\frac{223}{175}\right)^{-14.5} = ?$

1. First divide 223 by 175 and verify that expression becomes $(1.274)^{-14.5}$.
2. Now verify that $(1.274)^{14.5}$ is located on LL3 and, hence, $(1.274)^{-14.5}$ must be on LLo3. Result is ***.030.***

The foregoing illustrate the general procedure:

> *To evaluate b^x (x negative):*
> Proceed as if to evaluate b^x *(x positive)*, then read the result on the *reciprocal* LL scale.

Exercise 20-3

1. $6^{-4} =$

2. $3^{-5} =$

3. $(1.5)^{-7} =$

4. $(1.18)^{-2.5} =$

5. $(0.75)^{-3} =$

6. $(0.60)^{-5} =$

7. $(0.975)^{-30} =$

8. $(1.027)^{-18} =$

9. $(.006)^{-.07} =$

10. $(0.16)^{-.03} =$

11. $(264)^{-0.33} =$

12. $(.034)^{-0.26} =$

13. $(3700)^{-.025} =$

14. $(0.465)^{-.15} =$

15. $(56)^{-.252} =$

16. $(1.0276)^{-19.5} =$

17. $(.000075)^{-.064} =$

18. $(5600)^{-.007} =$

19. $(0.753)^{-8.2} =$

20. $(0.9783)^{-0.56} =$

21. $(.0575)^{-0.133} =$

22. $(.00013)^{-0.48} =$

23. $(0.9644)^{-28} =$

24. $(0.856)^{-52} =$

25. $3400(1 + 0.105)^{-14} =$

26. $25{,}000(1 + .035)^{-21} =$

27. $64{,}000(1 + .065)^{-34} =$

28. $\left(\frac{74}{15}\right)^{-.15} =$

29. $\left(\frac{465}{680}\right)^{-16} =$

30. $\left(\frac{6.9}{325}\right)^{-.072} =$

20.6 Evaluating b^x on slide rules with 8 LL scales

If your slide rule has 8 LL scales, you may check the following examples:

Example 1: $(1.00236)^{240} = ?$

1. Move HL over 1.00236 on LL0 (Ln0).
2. Slide left index of C under HL.
3. Move HL over 24 on C. Answer is located 2 scales higher on LL2.
4. Under HL read ***1.76*** on LL2.

Example 2: $(250)^{-.00065} = ?$

1. Move HL over 250 on LL3. Slide right index of C under HL.
2. Move HL over 65 on C. If exponent were positive, answer would be located 3 scales lower on LL0 (Ln0); exponent is negative, hence answer is on the reciprocal scale, LLo0 (Ln-0, LL/0).
3. Under HL read ***0.99642*** on LLo0 (Ln-0, LL/0).

Example 3: $(0.99805)^{1500} = ?$

1. Move HL over 0.99805 on LLo0 (Ln-0, LL/0). Slide left index of C under HL.
2. Move HL over 15 on C. Answer is located 3 scales higher on LLo3 (Ln-3, LL/3).
3. Under HL read ***.0536*** on LLo3 (Ln-3, LL/3).

Verify the following (use LL0 and LLo0 scales):

1. $(1.00164)^{30} = 1.0504$
2. $(1.00425)^{-820} = .0310$
3. $(1.223)^{.032} = 1.00646$
4. $(.0034)^{.00054} = 0.99694$
5. $(0.9763)^{-0.176} = 1.00423$
6. $(0.998745)^{4600} = .0031$
7. $(3200)^{.000185} = 1.001495$
8. $(0.99652)^{-23} = 1.0835$

20.7 Finding $b^{1/x}$

Example 1: $\sqrt[3]{72} = (72)^{1/3} = ?$

The logarithm may be written:

$$\ln(72)^{1/3} = (1/3)\ln(72)$$

In this case, then, we must *divide* ln(72) by 3. Note, also, that the exponent is *approximately 0.3.*

1. Move HL over 72 on LL3. This puts HL over ln(72) on D. Now divide by 3:
2. Slide 3 on C under HL.
3. Move HL over left index of C. HL is to the left of 72; hence, if exponent were about 3, answer would be on "LL4." Decimal point in exponent is actually one place to the left; therefore, answer is one scale lower on LL3.
4. Under HL read ***4.16*** on LL3.

Example 2: $\sqrt[12]{2.93} = (2.93)^{1/12} = ?$

Note that exponent is *about .08.*

1. Move HL over 2.93 on LL3. HL is now over ln(2.93) on D. Now *divide* by 12:
2. Slide 12 on C under HL.
3. Move HL over right index of C. HL is to the right of 2.93; hence, if exponent were about 8, answer would be on LL3. Decimal point in exponent is actually two places to the left of this position; therefore, answer is two scales lower on LL1.
4. Under HL read ***1.0938*** on LL1.

Example 3: $(.0115)^{1/60} = ?$

Note that exponent is *about .016.*

1. Move HL over .0115 on LLo3. Now *divide* by 60:
2. Slide 60 on C under HL.
3. Move HL over right index of C. HL is to right of .0115; hence, if exponent were about 1.6, answer would be on LLo3. Decimal point is actually two places to the left of this position; hence, answer is two scales lower on LLo1.
4. Under HL read ***0.9283*** on LLo1.

Example 4: $(0.9804)^{-1/.045} = ?$

Note that exponent is about −20.
Verify that result is on LL2, and is equal to ***1.553.***

Verify the following:

1. $\sqrt[4]{520} = 4.79$
2. $\sqrt[3]{2.64} = 1.382$
3. $\sqrt[12]{.0035} = 0.624$
4. $\sqrt[6]{0.550} = 0.9052$
5. $(1.345)^{1/4.4} = 1.0696$
6. $(0.243)^{-1/22} = 1.0665$
7. $(0.9605)^{1/.035} = 0.316$
8. $(15.5)^{-1/70} = 0.9616$

Example 5: Evaluate $10^{1/x}$ for $x = 3, 4, 5,$ and 6.

Here, it is more convenient to perform the consecutive divisions using the CI scale.

1. Move HL over 10 on LL3.
2. Slide left index of C under HL.
3. Move HL successively over 3, 4, 5, and 6 on CI.

Verify that results are: ***2.155, 1.779, 1.585,*** and ***1.468.***

Example 6: $x^{2.3} = 36; x = ?$

1. Solving for x: $x = (36)^{1/2.3}$.
2. Verify that $x =$ ***4.75.***

Example 7: $25(y^{.084}) = 47; y = ?$

1. Dividing 47 by 25, we obtain: $y^{.084} = 1.88$.
2. Solving for y: $y = (1.88)^{1/.084}$.
3. Verify that $y =$ ***1830.***

Exercise 20-4

1. Evaluate $(32)^{1/x}$ for $x = 3, 5, 9, 15$.
2. $\sqrt[3]{275} =$
3. $\sqrt[3]{5.61} =$
4. $\sqrt[3]{.037} =$
5. $\sqrt[4]{2.55} =$
6. $\sqrt[4]{2200} =$
7. $\sqrt[5]{0.145} =$
8. $\sqrt[5]{55} =$
9. $\sqrt[7]{1.95} =$
10. $\sqrt[7]{0.474} =$
11. $\sqrt[6]{0.636} =$
12. $(12.5)^{1/55} =$
13. $(6.75)^{1/1.6} =$
14. $(0.515)^{-1/25} =$
15. $(.00014)^{1/2.2} =$
16. $(0.944)^{-1/.045} =$
17. $(145)^{1/15} =$
18. $(1.026)^{1/.024} =$
19. $(1.0245)^{-1/.007} =$
20. $\sqrt[10]{0.764} =$
21. $(2000)^{1/24} =$
22. $(0.9656)^{1/.032} =$
23. $(360)^{1/17} =$
24. $x^{2.2} = 1.7$; $x =$
25. $x^{0.3} = 5$; $x =$
26. $x^{15} = 0.78$; $x =$
27. $x^{1.75} = 0.575$; $x =$
28. $x^{-.053} = 0.9724$; $x =$
29. $2.76(x^{3.5}) = 400$; $x =$
30. $15(x^{-31}) = 3900$; $x =$
31. $340(x^{37.5}) = 0.289$; $x =$
32. $2400(x^{-3.56}) = 1740$; $x =$
33. $\left(\frac{56}{11}\right)^{1/7} =$
34. $\left(\frac{475}{620}\right)^{-1/.055} =$
35. $\left(\frac{2.4}{115}\right)^{1/14} =$
36. $\left(\frac{1.570}{.0042}\right)^{1/240} =$
37. $37.5\left(\frac{125}{340}\right)^{1/1.41} =$
38. $216\left(\frac{87.5}{31.0}\right)^{1/1.41} =$

20.8 More general powers of b

Example 1: $(1.055)^{25\times 2.8} = ?$

When the exponent is in combined form, we may first evaluate the exponent on C·D, then find the power. You may prefer this direct approach; however, in the following precedure you will note that it is not really necessary to evaluate the exponent.

1. Approximate the exponent to be about 70. Think of the logarithm: $\ln(1.055)^{25\times2.8} = (25 \times 2.8)\ln(1.055)$.
2. Move HL over 1.055 on LL1. This puts HL over ln(1.055) on D. Now, instead of multiplying by 25, divide by its reciprocal:
3. Slide 25 on CI under HL. Now multiply by 2.8:
4. Move HL over 28 on C. Observing that HL is to the left of 1.055 with the exponent about 70, we determine that the answer must be on LL3.
5. Under HL read ***42.3*** on LL3.

Example 2: $(4)^{5/3} = ?$

1. Exponent is about 1.7. Think of the logarithm: $\ln(4)^{5/3} = (5/3)\ln(4)$.
2. Move HL over 4 on LL3. This puts HL over ln(4) on D. Now divide by 3:
3. Slide 3 on C under HL. Now multiply by 5:
4. Move HL over 5 on C. HL is to the right of 4 and exponent is about 1.7; hence, answer is on LL3.
5. Under HL read ***10.1*** on LL3.

Example 3: $(2.06)^{-(5.6\times1.3)/155} = ?$

1. Approximate exponent to be about −.05.
2. Move HL over 2.06 on LL2. Slide right index of C under HL. Now multiply by 5.6:
3. Move HL over 56 on C. Now divide by 155:
4. Slide 155 on C under HL. Now multiply by 1.3:
5. Move HL over 13 on C. Observing that HL is to the left of 2.06, verify that, if exponent were positive, answer would be on LL1. It follows that answer must be on LLo1.
6. Under HL read ***0.9666*** on LLo1.

Example 4: $(.0145)^{1/(31\times4.6)} = ?$

1. Approximate exponent to be about .007.
2. Verify that answer is located on LLo1, and is equal to ***0.97075.***

The techniques discussed in the preceding sections may finally be summarized as follows:

> *To raise b to any power:*
> 1. If exponent is in reciprocal or combined form, estimate its value. This aids in locating the result on the proper LL scale.
> 2. Move HL over b on the LL scale. This puts HL over ln(*b*) on D.
> 3. Perform necessary operations on ln(*b*) – that is:
>   multiplication (b^x, b^{xy}, etc.),
>   division ($b^{1/x}$, $b^{1/xy}$, etc.),
>   or both ($b^{x/y}$, $b^{xy/z}$, $b^{x/yz}$, etc.).
> 4. Read result on the appropriate LL scale.

Exercise 20-5

1. $(11)^{4/3} =$
2. $(8)^{5/7} =$
3. $(1.6)^{5/6} =$
4. $(36)^{-2/7} =$
5. $(0.7)^{4/9} =$
6. $(0.944)^{-13/7} =$
7. $(.034)^{-3/460} =$
8. $(1.0374)^{52 \times 0.34} =$
9. $(2.62)^{5.1 \times 1.8} =$
10. $(27.5)^{-(3.2 \times 4.7)/23} =$
11. $(.074)^{1/(4.5 \times 16.3)} =$
12. $(0.284)^{-100/(5.2 \times 3.7)} =$
13. $(4400)^{(3.7 \times 1.45)/34.6} =$
14. $(.00028)^{3/40} =$
15. $(0.9025)^{.064 \times 27} =$
16. $(1.0186)^{-100/(2.45 \times 3.14)} =$
17. $(650)^{(36 \times 2.7)/1250} =$
18. $(1.108)^{-(530 \times .077)} =$
19. $(.0135)^{2.24/(145 \times 1.95)} =$
20. $(0.9786)^{-68/\pi} =$
21. $(8.4)^{42/65} =$
22. $(1.0742)^{-(\pi \times 36)/2.3} =$
23. $x^{5/7} = 23$; $x =$
24. $x^{9/4} = 1.26$; $x =$
25. $36(x^{15/4}) = 250$; $x =$
26. $2.3(x^{1/6}) = 1.7$; $x =$
27. $x^{-5/16} = 0.955$; $x =$
28. $14(x^{-36/\pi}) = 175$; $x =$
29. $(1 + x)^{200/7} = 1.85$; $x =$
30. $(1 + x)^{-16/9} = 0.215$; $x =$
31. $360(1 - x)^{-25/11} = 570$; $x =$
32. $\left(1 + \frac{23}{670}\right)^{-4/.015} =$
33. $\left(1 - \frac{15}{463}\right)^{-240/13} =$
34. $624 \left(\frac{6.4}{2.7}\right)^{0.415/1.415} =$
35. $545 \left(\frac{34.2}{86.5}\right)^{0.415/1.415} =$
36. $\frac{575}{405} = \left(\frac{P}{50}\right)^{0.384/1.384}$; $P =$
37. $\frac{425}{630} = \left(\frac{P}{7.4}\right)^{0.400/1.400}$; $P =$
38. $\frac{560}{415} = \left(\frac{28.5}{P}\right)^{0.375/1.375}$; $P =$
39. Evaluate $(0.98)^{-p/1.75}$ for: $p = 1, 12, 150,$ and 750.

20.9 Numbers outside the range of the LL scales.

The exercise sets of this chapter have been designed so that the initial settings and the results have, in all cases, been within the LL scale range. Methods for handling numbers outside the scale range are illustrated in Appendix B.

Chapter 21

FURTHER OPERATIONS WITH THE LL SCALES

21.1 Solving for the exponent in simple powers

In the previous chapter, we were concerned with evaluating powers directly. It often happens, however, that the power is known and we wish to find the exponent.

Example 1: $6^x = 1.55$; $x = ?$

Here, we know the result of raising 6 to a power, and we must determine the exponent. This is just the inverse of finding b^x.

1. Move HL over 6 on LL3. Slide left index of C under HL.
2. Move HL over 1.55 on LL2. Under HL read "245" on C. Observe that 1.55 is to the right of 6 and one scale lower. If it were on the same scale, exponent would be 2.45; however, it is one scale lower, hence exponent must be 0.245. It follows that $x =$ ***0.245.***

Example 2: $(0.955)^x = .064$; $x = ?$

1. Move HL over 0.955 on LLo1. Slide right index of C under HL.
2. Move HL over .064 on LLo3. Under HL read "596" on C. Note that .064 is to the left of 0.955 and is two scales higher. If it were one scale higher, exponent would be 5.96; however, it is two scales higher, hence $x =$ ***59.6.***

Example 3: $(1.22)^x = .028$; $x = ?$

1. Move HL over 1.22 on LL2. Slide left index of C under HL.

2. Move HL over .028 on LLo3. Under HL read "1800" on C. Note that .028 is to the right of 1.22 on LLo3. If it were on LL3, exponent would be 18.00; however, it is on the *reciprocal* LL scale, hence $x = $ ***–18.00.***

Example 4: $(.0165)^x = 1.022$; $x = ?$

1. Move HL over .0165 on LLo3. Slide right index of C under HL.
2. Move HL over 1.022 on LL1. Under HL read "530" on C. Observe that 1.022 is to the left of .0165 on LL1. If it were on LLo1, exponent would be .00530; however, it is on the reciprocal LL scale, hence $x = $ ***–.00530.***

Example 5: $e^x = 1.0466$; $x = ?$

When e is the base, we read directly from LL to D.

1. Move HL over 1.0466 on LL1.
2. Under HL read "455" on D. On LL1, exponent of e ranges from .01 to 0.1, hence $x = $ ***.0455.***

Example 6: $e^{-x} = .0033$; $x = ?$

1. Move HL over .0033 on LLo3.
2. Under HL read "571" on D. On LLo3, exponent of e ranges from –1 to –10. It follows that $-x = -5.71$, and $x = $ ***5.71.***

Exercise 21-1

Determine x in the following:

1. $3^x = 20$
2. $10^x = 250$
3. $8^x = 54$
4. $4^x = 30$
5. $25^x = 7$
6. $(1.2)^x = 100$
7. $(1.03)^x = 200$
8. $e^x = 30$
9. $e^x = 550$
10. $e^x = 1.266$
11. $e^x = 0.9718$
12. $e^x = .00085$
13. $10^x = 1.34$
14. $(0.16)^x = .012$
15. $10^x = .02$
16. $5^x = 0.64$
17. $525^x = 7.5$
18. $(0.75)^x = 0.464$

19. $15^x = 0.65$

20. $(2.4)^x = 650$

21. $(1.165)^x = 1.0375$

22. $(1.84)^x = 0.766$

23. $(.037)^x = 0.49$

24. $(0.164)^x = 1.0525$

25. $(0.654)^x = .0475$

26. $e^x = 1.01675$

27. $e^{-x} = 0.562$

28. $e^{-x} = 0.106$

29. $10^x = 0.57$

30. $(2600)^{-x} = 0.124$

31. $(1.0164)^x = 64$

32. $e^{-x} = 0.9326$

33. $e^x = 1650$

34. $(0.586)^x = .00075$

35. $(3200)^{-x} = 0.794$

36. $(1.238)^{-x} = 0.513$

37. $(1.1645)^x = .0037$

38. $(0.386)^x = .084$

39. $(0.193)^x = 1.775$

40. $(1250)^x = 1.0138$

21.2 Finding logarithms to any base

Suppose we let $\log_b N = x$; it follows that $b^x = N$. Hence, finding $\log_b N$ leads to the same exponential form that was treated in the preceding section.

Example 1: $\log_2 5 = ?$

1. Let $\log_2 5 = x$; then $2^x = 5$.
2. Verify that $x = 2.32$; hence, $\log_2 5 =$ ***2.32.***

Example 2: $\log_3(0.62) = ?$

1. Let $\log_3(0.62) = x$; then $3^x = 0.62$.
2. Verify that $x = -0.435$; hence, $\log_3(0.62) =$ ***−0.435.***

Example 3: $\log_{10} 1.06 = ?$

1. Let $\log_{10} 1.06 = x$; then $10^x = 1.06$.
2. Verify that $x = .0253$; hence, $\log_{10} 1.06 =$ ***.0253.***

You will recall that we may find $\log_{10} N$ directly using the L scale. However, for values of N near 1, the LL scales yield more accuracy.

21.3 The DFM scale

The DFM scale is a D scale folded at $\log_{10}e = 0.434$. It is related to the D scale in the following way.

> If the hairline is over *ln N* on **D**, then $log_{10}N$ is under the hairline on **DFM**

This scale makes it possible to directly convert logs from base *e* to base 10, and vice versa.

Example: **a.** ln 26 = ? **b.** $\log_{10}26$ = ?

1. Move HL over 26 on LL3.
2. Under HL read:
  a. ln 26 = ***3.26*** on D.
  b. $\log_{10}26$ = ***1.415*** on DFM.

Exercise 21-2

1. $\log_2 7$ =

2. $\log_4 18$ =

3. $\log_8 35$ =

4. $\log_3 1.5$ =

5. $\log_{2.5} 100$ =

6. $\log_5 1.75$ =

7. $\log_{10} 1.155$ =

8. $\log_{10} 0.984$ =

9. $\log_{10} 0.9723$ =

10. $\log_{10} 1.01655$ =

11. $\log_{10} 1.0224$ =

12. $\log_3 0.635$ =

13. $\log_6 .0846$ =

14. $\log_{12} 350$ =

15. $\log_{7.5} 0.77$ =

16. $\log_{20} 1.27$ =

17. $\log_{10} 1.0444$ =

18. $\log_9 175$ =

19. $\log_{1.2} 1.88$ =

20. $\log_{2.6} 0.94$ =

21. $\log_8 4500$ =

22. $\log_{3.8} 0.665$ =

23. $\log_{10} 0.8645$ =

24. $\log_7 .0035$ =

21.4 Exponential equations

Any equation in which the unknown appears in an exponent may be called an exponential equation. In section 21.1, you were solving simple exponential equations; in this section we go further with equations of this type.

Example 1: $35e^{-.22t} = 0.106;\ t = ?$

1. Dividing 0.106 by 35 on the C-D scales, we obtain:

$$e^{-.22t} = .00303$$

2. Move HL over .00303 on LLo3.
3. Under HL read "580" on D. Therefore, $-.22t = -5.80$.
4. Dividing on C-D, we find $t =$ ***26.4.***

Example 2: $125(370)^x = 90;\ x = ?$

1. Dividing 90 by 125 on C-D, we obtain:

$$370^x = 0.720$$

2. Move HL over 370 on LL3. Slide right index of C under HL.
3. Move HL over 0.720 on LLo2. Under HL read "556" on C. Note that 0.720 is to the left of 370 on LLo2. If it were on LL2, exponent would have to be .0556; however, it is on the *reciprocal* LL scale, hence $x =$ ***−.0556.***

Example 3: $(0.73)^{.034x} = 0.865;\ x = ?$

1. Move HL over 0.73 on LLo2. Slide right index of C under HL.
2. Move HL over 0.865 on LLo2. Under HL read "461" on C. Note that 0.865 is to the left of 0.73 and on the same scale, hence exponent must be 0.461. It follows that $.034x = 0.461$.
3. Dividing on C-D, we obtain $x =$ ***13.56.***

Example 4: $75 = 1000(1.85)^{-14k};\ k = ?$

1. Dividing 75 by 1000, we may write:

$$(1.85)^{-14k} = .075$$

2. Move HL over 1.85 on LL2. Slide right index of C under HL.
3. Move HL over .075 on LLo3. Under HL read "421" on C. Note that .075 is to the left of 1.85 on LLo3. If it were on LL3, exponent would be 4.21; however, it is on the reciprocal LL scale, hence exponent must be −4.21. Therefore, $-14k = -4.21$.
4. Dividing on C-D, we obtain $k =$ ***0.301.***

Alternate method:
Although more settings are required, you may prefer to first solve the given relation for k.

1. Returning to $(1.85)^{-14k} = .075$, we take natural log of both sides:

$$(-14k)\ln(1.85) = \ln(.075)$$

2. Solving for k:

$$k = \frac{\ln(.075)}{(-14)(\ln(1.85)}$$

3. Verify that $\ln(.075) = -2.59$, and $\ln(1.85) = 0.615$.
4. Substituting these values, k may be evaluated on C-D:

$$k = \frac{-2.59}{(-14)(0.615)} = \boldsymbol{0.301}.$$

Exercise 21-3

1. $e^{12x} = 1550;\ x =$

2. $e^{.0065x} = 1.0665;\ x =$

3. $e^{-10x} = 0.374;\ x =$

4. $e^{-.024x} = .0028;\ x =$

5. $3.5\ e^{x} = 460;\ x =$

6. $125\ e^{x} = 60;\ x =$

7. $550\ e^{-x} = 30;\ x =$

8. $e^{.0053x} = 1.267;\ x =$

9. $e^{k/12.5} = 1.0326;\ k =$

10. $4.63\ e^{3.6k} = 2150;\ k =$

11. $2000\ e^{-.064t} = 50;\ t =$

12. $1200\ e^{-.0075t} = 125;\ t =$

13. $10.7\ e^{75k} = 145{,}000;\ k =$

14. $100\ e^{-2\pi k} = 5.5;\ k =$

15. $26\ e^{t/16} = 94{,}000;\ t =$

16. $15{,}000\ e^{-t/25} = 220;\ t =$

17. $2.5(1.745)^{x} = 362;\ x =$

18. $6500(0.815)^{x} = 20.8;\ x =$

19. $24.6(0.586)^{-x} = 45.0;\ x =$

20. $1000(1.0625)^{n} = 3640;\ n =$

21. $2200(1.125)^{n} = 12{,}600;\ n =$

22. $1000(1.075)^{n} = 525;\ n =$

23. $8600(1.045)^{n} = 2700;\ n =$

24. $\left(\frac{12.6}{41.5}\right)^{x} = \frac{550}{640};\ x =$

25. $\left(\frac{34.5}{57.2}\right)^{x} = \frac{440}{510};\ x =$

26. $(1.074)^{-1.7k} = 0.948;\ k =$

27. $(840)^{.037x} = 1.056;\ x =$

28. $(250)^{-\pi x} = 0.744;\ x =$

29. $(0.162)^{.0045p} = 0.935;\ p =$

30. $(6000)^{.072t} = 64;\ t =$

31. $0.27(1.055)^{360t} = 50;\ t =$

32. $324(4.16)^{2\pi k} = 340;\ k =$

33. $3.6(1.15)^{.045t} = 31.0;\ t =$

34. $145(2.35)^{-.018t} = 26;\ t =$

35. $225(0.65)^{k/32} = 1.52;\ k =$

36. $1200(.00055)^{.0045p} = 1170;\ p =$

37. $0.410(26.5)^{-22x} = 0.336;\ x =$

38. $55(7500)^{-.0135t} = 24.2;\ t =$

39. $\frac{510}{620} = \left(\frac{8.25}{31.6}\right)^{(n-1)/n};\ n =$

40. $\frac{435}{520} = \left(\frac{126}{147}\right)^{(n-1)/n};\ n =$

41. $2e^{2x} - 12e^{x} + 5 = 0;\ x =$

42. $3e^{2x} - 8e^{x} + 3 = 0;\ x =$

43. $2e^{6x} - 31e^{3x} + 58 = 0;\ x =$

44. $12e^{-2x} - 11e^{-x} + 2 = 0;\ x =$

21.5 Formula types

Example 1: $\frac{(1 + .063\sqrt[3]{75})^{5.8}}{(0.65)^{3.2}} = ?$

1. Verify that $.063\sqrt[3]{75} = 0.266$
2. Expression now becomes: $\frac{(1.266)^{5.8}}{(0.65)^{3.2}}$.
3. Verify that $(1.266)^{5.8} = 3.93$; $(0.65)^{3.2} = 0.252$.
4. Expression finally becomes: $\frac{3.93}{0.252}$.
5. Dividing on C-D, verify that result is ***15.59.***

Example 2: Given the formula: $S = R\left[\frac{(1+i)^n - 1}{i}\right]$

If $S = 7500$, $R = 37$, $i = .035$, find n to the nearest integral value.

1. Substitute the given values:

$$7500 = \frac{37\,[(1.035)^n - 1]}{.035}$$

2. Verify that $(1.035)^n = \frac{.035 \times 7500}{37} + 1 = 7.1 + 1 = 8.1$.
3. Finally verify that $n = 60.8$; hence, to nearest integer, $n =$ ***61.***

Exercise 21-4

1. $\sqrt{29(1 + e^{-1.84})} =$

2. $15(0.32)^4(0.68)^2 =$

3. $120(0.72)^7(0.28)^3 =$

4. $\dfrac{7500}{1 + 12.5e^{-2.26}} =$

5. $.046e^{-2.7\pi} \sin 1.75 =$

6. $3.66 \times 10^7 \times \ln\left[\dfrac{6.44 - 1.27}{6.44 + 1.27}\right] =$

7. $\dfrac{1.32 \times 10^{-9} \times \sqrt{870}}{(.072)^{5/4}} =$

8. $\dfrac{(250)^{0.76}}{(25 + 12\sqrt{4.65})^{1.16}} =$

9. $\ln\left[\dfrac{1 + \sqrt{1 + (2.6)^2}}{2.6}\right] =$

10. $\dfrac{240\,[(1.0275)^{11} - 1]}{.0275} =$

11. $35\left[\dfrac{1 - (1.045)^{-15}}{.045}\right] =$

12. $\dfrac{10!}{8!2!}(0.84)^8(0.16)^2 =$

13. $\dfrac{10!}{6!4!}(0.43)^6(0.57)^4 =$

14. $(1260 \cos 53°)^{0.6}(0.164)^{2/3}(136)^{1/4} =$

15. $\dfrac{1670 \times 0.155}{(1 + 0.155)^{1/6} - 1} =$

16. $4.25 \times 10^6[50 + \pi/.065]^{1/23} =$

17. $67{,}500(0.86)^{5/2}\tan 62° =$

18. $[1 + .000275(27.2)^2]^{1.75/0.75} =$

19. $\dfrac{5.66 \times 10^{-17}}{\ln 7.24}[1 + \sqrt{7.25}] =$

20. $(0.72)^{-5}\,[e^{\pi/(0.72 \times 45)} - 1]^{-1} =$

In the following formulas, substitute the given data and evaluate:

21. $P = \dfrac{9.2\,D^{1.7}}{R + 5}$
a. $R = 3.75$, $D = 8$; **b.** $R = 8.60$, $D = 14.5$

22. $N = e^{2\pi d/\sqrt{1-d^2}}$
a. $d = 0.72$; **b.** $d = 0.46$

23. $x = \ln(u + \sqrt{u^2 + 1})$
a. $u = 2.20$; **b.** $u = -0.176$

24. $y = \dfrac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$
a. $x = 0.25$; **b.** $x = 0.5$; **c.** $x = 1$; **d.** $x = 2.50$

25. $X = e^{-2.44t} \sin 0.46t$ (angle in radians)
a. $t = 1$; **b.** $t = 2.5$; **c.** $t = 5.7$

26. $V = 1.318\,C\,R^{0.63}\,S^{0.54}$
a. $C = 130$, $R = 0.25$, $S = .003$
b. $C = 750$, $R = .048$, $S = .0076$

27. $E = RT\ln(p_1/p_2)$
a. $R = 55.1$, $T = 500$, $p_1 = 54.7$, $p_2 = 36.0$
b. $R = 29.5$, $T = 350$, $p_1 = 24.2$, $p_2 = 18.6$

28. $A = \dfrac{250}{1 + 16e^{-.023t}}$

a. $t = 75$; **b.** $t = 125$

29. $p = \dfrac{b^x e^{-b}}{x!}$

a. $b = 0.2, x = 5$; **b.** $b = 2, x = 4$

30. $F_1 = F_2 e^{f\Theta}$ (Θ in radians).

a. $f = 0.25$, $\Theta = 210°$, $F_2 = 1800$

b. $f = 0.17$, $\Theta = 104°$, $F_2 = 2400$

31. $h = (p_0/w_0)\ln(p_0/p)$

a. $p_0 = 1830$, $w_0 = .0765$, $p = 1250$

b. $p_0 = 2620$, $w_0 = 1.140$, $p = 1780$

32. $Q = 2.38\, H^{5/2} \tan \dfrac{1}{2}\Theta$

a. $H = 3.6$, $\Theta = 70°$; **b.** $H = 6.2$, $\Theta = 55°$

33. $PV^{1.4} = 16000$

a. $V = 27.8$, find P; **b.** $V = 460$, find P; **c.** $P = 760$, find V;
d. $P = 1240$, find V.

34. $C = \dfrac{6}{\pi \ln(d_1/d_2)} \left[\dfrac{(d_1/d_2) - 1}{2}\right]$

a. $d_1 = 11.2$, $d_2 = 6.7$; **b.** $d_1 = 34.7$, $d_2 = 14.6$

35. $P = 650{,}000\, e^{kt}$

a. If $P = 925{,}000$ when $t = 10$, evaluate k.

b. Use this value of k to find P when $t = 18$.

36. $E = 14.2\, e^{-kt}$

a. If $E = 10.2$ when $t = 3$, evaluate k.

b. Use this value of k to find E when $t = 6.3$.

37. $P = \dfrac{N!}{x!(N-x)!} p^x q^{N-x}$

a. $N = 10$, $x = 4$, $p = 0.59$, $q = 0.41$

b. $N = 10$, $x = 7$, $p = 0.81$, $q = 0.19$

c. $N = 9$, $x = 3$, $p = 0.44$, $q = 0.56$

d. $N = 11$, $x = 5$, $p = 0.65$, $q = 0.35$

38. $A = P(1 + i)^n$

a. $P = 1000$, $A = 2250$, $n = 21$, find i;

b. $P = 1200$, $A = 2100$, $n = 12$, find i;

c. $P = 1200$, $A = 3000$, $i = .01625$, find n (nearest integral value);

d. $P = 650$, $A = 3500$, $i = .0275$, find n (nearest integral value).

39. $S = R\left[\dfrac{(1+i)^n - 1}{i}\right]$

a. $R = 55$, $i = .035$, $n = 15$; **b.** $R = 75$, $i = .0625$, $n = 22$;
c. $S = 2800$, $R = 75$, $i = .025$, find n (nearest integral value);
d. $S = 20{,}000$, $R = 825$, $i = .018$, find n (nearest integral value).

40. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

a. $A = 1150$, $R = 63$, $i = .0325$, find n (nearest integral value);
b. $A = 7250$, $R = 215$, $i = .0125$, find n (nearest integral value).

41. $P = Fr/i + (V - Fr/i)(1+i)^{-n}$

a. $F = 2000$, $V = 2600$, $r = .045$, $i = .0625$, $n = 9$
b. $F = 15{,}000$, $V = 18{,}500$, $r = .035$, $i = .0825$, $n = 12$

42. $\dfrac{T_1}{T_2} = \left(\dfrac{P_1}{P_2}\right)^{(n-1)/n}$

a. $P_1 = 14$, $P_2 = 73$, $T_2 = 550$, $n = 1.415$, find T_1;
b. $P_1 = 15$, $P_2 = 107$, $T_2 = 635$, $n = 1.346$, find T_1;
c. $T_1 = 520$, $T_2 = 625$, $P_1 = 15.5$, $P_2 = 39$, find n;
d. $T_1 = 575$, $T_2 = 740$, $P_1 = 19$, $P_2 = 42$, find n.

43. $V = V_f \tanh\left(\dfrac{gHt}{V_f L}\right)$

a. $H = 20$, $t = 10$, $V_f = 4.55$, $L = 2000$, $g = 32.2$
b. $H = 30$, $t = 12$, $V_f = 4.75$, $L = 1800$, $g = 32.2$

44. $S = 2\pi a^2 + \dfrac{\pi b^2}{e}\ln\left(\dfrac{1+e}{1-e}\right)$ (where $e = \sqrt{a^2 - b^2}/a$)

a. $a = 12$, $b = 7$; **b.** $a = 26.5$, $b = 18.3$

45. $E = \dfrac{p_1}{w_1}\left[\dfrac{k}{k-1}\right]\left[1 - \left(\dfrac{p_2}{p_1}\right)^{(k-1)/k}\right]$

a. $k = 1.32$, $p_1 = 2880$, $p_2 = 2120$, $w_1 = .0613$
b. $k = 1.55$, $p_1 = 3150$, $p_2 = 2120$, $w_1 = .0580$

46. $r = \left[1 + \left(\dfrac{k-1}{2}\right)\left(\dfrac{V}{c}\right)^2\right]^{k/(k-1)}$

a. $k = 1.4$, $V = 576$, $c = 1117$
b. $k = 1.65$, $V = 645$, $c = 1117$

Chapter 22

REVIEW EXERCISES

The following exercise sets require the use of all scales covered in Chapters 14 through 21.

Exercise 22-1

1. $\sin 37° =$
2. $\cos 86°30' =$
3. $\tan 47.5° =$
4. $\log_{10}38.2 =$
5. $e^{0.134} =$
6. $e^{-.075} =$
7. $(1.075)^{27} =$
8. $(260)^{-0.32} =$

Exercise 22-2

1. Solve the right triangle: $A = 46.2°$, $b = 370$.
2. $\ln 26.2 =$

3. $(13)^{5/3} =$

4. $\cot 14.6° =$

5. $2600e^{.046} =$

6. $15(x^{.075}) = 27;\ x =$

7. $24 \sin 36° =$

8. $\sqrt[5]{0.138} =$

Exercise 22-3

1. $\dfrac{145 \cos 62°}{\sin 27°} =$

2. $\ln 1.0137 =$

3. $e^{-3.27} =$

4. $\tan 3.45° =$

5. $\left(\dfrac{6.22}{21.5}\right)^{0.57} =$

6. $\sin^{-1} 0.644 =$

7. $(520)^x = 4.25;\ x =$

8. Solve the oblique triangle: $a = 82$, $c = 55$, $A = 43°$.

Exercise 22-4

1. $(4400)^{.0026} =$

2. $\cos 17° 23' =$

3. $\tan^{-1} 1.25 =$

4. $e^{0.62} \sin 22° =$

5. $\log_{10} 56{,}500 =$

6. $\dfrac{\ln .0245}{\pi} =$

7. Change $24\angle 56^\circ$ to rectangular form.

8. $\log_3 8 =$

Exercise 22-5

1. $(7500)^{.035} =$

2. $.0375 \log_{10}\left[\frac{16.25}{.00352}\right] =$

3. $e^{-0.455} \cos 0.62 =$

4. $\ln N = 1.46$; $N = ?$

5. $640 \left(\frac{24.2}{37.5}\right)^{0.43/1.43} =$

6. Convert to radians: **a.** 36°, **b.** 76.3°, **c.** 214°.

7. $\sin 0.76^\circ =$

8. $2400\, e^{-.052t} = 75$; $t =$

Exercise 22-6

1. $\tan 257^\circ =$

2. $e^{-3\pi/2} =$

3. Change $(16 + 7j)$ to polar form.

4. $\frac{\ln 0.742}{.076 \ln 12.7} =$

5. $(32)^{-(3.4 \times 5.2)/310} =$

6. $175\, e^{-(3.6 \times 1.22)} \sin(0.72) =$

7. $\sin^{-1} .0356 =$

8. $\ln N = -3.25$; $N =$

Exercise 22-7

1. $\cos 88.36^\circ =$

2. $e^{(2.6\times 4.3)/7.5} =$

3. Solve the oblique triangle: $A = 124°$, $C = 13°$, $b = 21$.

4. $(0.15)^x = 0.14$; $x =$

5. $\sqrt[7]{350} =$

6. $\frac{1}{2}(16.8)(23.2)\sin 24°\ 15' =$

7. $45(0.78)^8(0.22)^2 =$

8. $\dfrac{\ln 3.25}{2.3 \ln 0.422} =$

Exercise 22-8

1. Solve the right triangle: $a = 145$, $b = 67$.

2. $\cos 0.68 =$

3. $(0.9780)^{-0.48} =$

4. $\dfrac{132 \sin^2 19.3°}{\sin 2.46°} =$

5. Given: $V = \sqrt{\dfrac{2gR}{\sin 2\Theta}}$
Find V if $R = 2400$, $g = 32.2$, $\Theta = 27.2°$.

6. $\sin^{-1}\left[\dfrac{12.8 + 8.44}{12.8 \times 8.44}\right] =$ (give answer in radians)

7. $\tan 86.5° =$

8. $165(2.75)^{-.022t} = 32$; $t =$

Appendix A

SMALL ANGLES (LESS THAN 0.573°)

A.1 Angles given in degrees

For angles in the range of the ST scale or smaller, the sine (or tangent) is proportional to the angle itself. Thus, if the angle is divided by 10, for example, its sine or tangent will also be divided by 10.

From the ST scale we may read sin 2.6° = .0454. Now, dividing both the sine and its angle by 10, we write: sin 0.26° = .00454. Clearly, we could have divided by 100 and obtained: sin .026° = .000454, and so forth.

It is emphasized that this procedure is only valid for very small angles; you should apply it only to angles that are too small to be read directly on the ST scale.

Example 1: sin .034° = ?

1. Move HL over 3.4° on ST.
2. Under HL read "593" on C.

Therefore, sin 3.4° = .0593, and sin .034° = ***.000593.***

Example 2: tan 89.22° = ?

1. Write: tan 89.22° = cot .08° = 1/tan .08°.
2. Verify that result is ***716.***

Verify the following:

1. sin 0.36° = .00628

2. sin .044° = .000767

3. tan 0.50° = .00872

4. tan .018° = .000314

5. tan 89.6° = 143.2

6. csc 0.15° = 382

A.2 Angles given in minutes or seconds

You may now verify that:

$$\sin 1' = \sin .01667° = .000291$$
$$\sin 1'' = \sin .000278° = .00000485$$

Since the sine (or tangent) of a small angle is proportional to the angle itself, the following relations may be used:

1. If angle is in *minutes*, then **sin x** or **(tan x)** = ***.000291 x.***
2. If angle is in *seconds*, then **sin x** or **(tan x)** = ***.00000485 x.***

The approximate values of these constants may be remembered as *"three zeros three"* and *"five zeros five."*

A.3 The "minutes" and "seconds" gauge marks

To facilitate finding the sine (or tangent) of small angles in minutes or seconds, some slide rules have two gauge marks related to the CI scale. These marks are scribed on the ST scale at positions opposite 291 and 485 on CI, and may be found at about 1.97° and 1.18°. They are usually labeled with the symbol for minutes (′) and the symbol for seconds (″) respectively.

On other rules, the marks are related to the C scale. They may be scribed directly on C at 291 and 485, or opposite these positions on ST at about 1.67° and 2.78°. Again, these are usually labeled with the minutes and seconds symbols.

The use of these gauge marks may be summarized as follows:

If gauge marks are related to CI:

1. Move HL over angle in minutes (or seconds) on D.
2. Slide "minutes" (or "seconds") scribe under HL.
3. Read sine (or tangent) of the angle opposite C index on D.

If gauge marks are related to C:

1. Set C index opposite angle in minutes (or seconds) on D.
2. Move HL over "minutes" (or "seconds") scribe.
3. Read sine (or tangent) of the angle under HL on D.

Example 1: sin 12.5′ = ?

A rough approximation is "three zeros three" times the angle, or .0003 × 12.5. Using the gauge marks, a more exact result is obtained.

Gauge marks related to CI:

1. Move HL over 125 on D.
2. Slide "minutes" scribe under HL.
3. Opposite right index of C read "364" on D.

From the rough approximation, we see that answer is ***.00364.***

Gauge marks related to C:

1. Set left index of C opposite 125 on D.
2. Move HL over "minutes" scribe.
3. Under HL read "364" on D. Answer is ***.00364.***

Example 2: tan 34″ = ?

A rough approximation is "five zeros five" times the angle, or .000005 × 34. Using the gauge mark, we proceed as follows:

Gauge marks related to CI:

1. Move HL over 34 on D.
2. Slide "seconds" scribe under HL.
3. Opposite left index of C read "1649" on D.

From the approximation, answer must be ***.0001649.***

Gauge marks related to C:

1. Set right index of C opposite 34 on D.
2. Move HL over "seconds" scribe.
3. Under HL read "1649" on D. Answer is ***.0001649.***

Verify the following:

1. sin 26′ = .00756
2. sin 7.5″ = .0000363
3. tan 14′30″ = .00422
4. sin 18.3′ = .00532
5. tan 20″ = .0000969
6. sin 3.7′ = .001077
7. tan 46″ = .000223
8. cot 25′18″ = 136
9. csc 12.4′ = 277

A.4 Conversion to radians

For the small angles we have been considering, the sine (or tangent) is about equal to the angle itself expressed in radians. Hence, to convert from degrees, minutes, or seconds, we simply find the sine of the angle and this also represents the radian equivalent of the angle.

Example 1: Convert 0.24° to radians.

1. Using the ST scale, verify that sin 0.24° = .00419.
2. It follows that 0.24° = ***.00419 radians.***

Example 2: Convert 25′ to radians.

1. Using the "minutes" gauge mark, verify that sin 25′ = .00727.
2. It follows that 25′ = ***.00727 radians.***

Verify the following:

1. 0.46° = .00802 radians

2. 37′ = .01076 radians

3. 28.5″ = .0001381 radians

4. .0375° = .000654 radians

5. 12.6′ = .00367 radians

6. 51.2″ = .000248 radians

7. 0.26° = .00454 radians

8. .071° = .00124 radians

Exercise A-1

1. sin 0.26° =

2. sin .077° =

3. tan .0145° =

4. sin .085° =

5. tan 89.7° =

6. cot 0.27° =

7. sin 0.46° =

8. tan 89.87° =

9. 1600 sin 0.4° =

10. 148 tan .065° =

11. Convert to radians:
a. 0.35° **b.** .071°
c. 0.185° **d.** .00225°

12. sin 5.8′ =

13. tan 12′15″ =

14. tan 35″ =

15. sin 2.44′ =

16. sin 13.5″ =

17. cot 22′30″ =

18. csc 11.25′ =

19. csc 33″ =

20. tan 89° 45.2′ =

21. tan 89° 59′26″ =

22. Convert to radians:
a. 43′ **b.** 26″
c. 8′40″ **d.** 11.5″

23. $\frac{1}{2}(170)(235)\sin 23' =$

24. $\frac{1}{2}(1350)(720)\sin 47'' =$

Appendix B

NUMBERS OUTSIDE THE RANGE OF THE LL SCALES

B.1 Numbers beyond the range of LL3 or LLo3

We have seen that the LL scales range from e^{-10} to e^{10}, or from .000045 to about 22,000. It is also evident that there is a discontinuity at the number 1; nowhere on the scale does this number appear. The scales approach 1 from above and from below, but never reach it. This is not surprising inasmuch as $1 = e^0$, and the number 0 does not appear on the D scale.

It follows that there are three ways in which we may fail to locate a number directly on the LL scales: the number may be *too large* (larger than 22,000), it may be *too small* (smaller than .000045), or it may be *too near 1*.

In this section we shall illustrate methods for dealing with numbers that are too large or too small (beyond the range of LL3 or LLo3). Numbers very near 1 are discussed in the next two sections.

Example 1: $(14)^5 = ?$

Rewrite this: $(1.4 \times 10)^5 = (1.4)^5 \times 10^5$.

Now, $(1.4)^5$ is within the range of the LL scale; hence, we may evaluate in the usual manner. Verify its value to be 5.38.

Therefore, result is $\boldsymbol{5.38 \times 10^5}$.

Example 2: $(.000032)^{-6} = ?$

Rewrite this: $(3.2 \times 10^{-5})^{-6} = (3.2)^{-6} \times 10^{30}$.

Evaluating in the usual manner: $(3.2)^{-6} = .00094$.

Hence, result is $.00094 \times 10^{30}$ or $\boldsymbol{9.4 \times 10^{26}}$.

Example 3: $(6350)^{5/2} = ?$

Rewrite this: $(0.635 \times 10^4)^{5/2} = (0.635)^{5/2} \times 10^{10}$.
Verify that $(0.635)^{5/2} = 0.322$.
Therefore, result is 0.322×10^{10} or $\boldsymbol{3.22 \times 10^9}$.

Example 4: $(5.6)^{-7.3} = ?$

Here, we divide the exponent by 2, then square the result.
Rewrite the expression: $[(5.6)^{-3.65}]^2$.
Verify that $(5.6)^{-3.65} = .00185$.
Result is $(.00185)^2 = (1.85 \times 10^{-3})^2 = \boldsymbol{3.42 \times 10^{-6}}$.

Example 5: $(155)^{14.3} = ?$

Rewrite this: $[1.55 \times 10^2]^{14.3} = (1.55)^{14.3} \times 10^{28.6}$.
This, in turn, may be written: $(1.55)^{14.3} \times 10^{0.6} \times 10^{28}$.
Verify that $(1.55)^{14.3} = 525$, and $10^{0.6} = 3.98$ (use L scale).
Result is $525 \times 3.98 \times 10^{28} = \boldsymbol{2.09 \times 10^{31}}$.

Example 6: $(.000425)^{7.22} = ?$

Rewrite this: $(0.425 \times 10^{-3})^{7.22} = (0.425)^{7.22} \times 10^{-21.66}$.
This may be written: $(0.425)^{7.22} \times 10^{0.34} \times 10^{-22}$.
Verify that $(0.425)^{7.22} = .0021$; $10^{0.34} = 2.19$ (use L scale).
Result is $.0021 \times 2.19 \times 10^{-22} = \boldsymbol{4.59 \times 10^{-25}}$.

Exercise B-1

1. $22^{4.6} =$

2. $e^{12} =$

3. $e^{-15} =$

4. $(1.032)^{500} =$

5. $12^{25} =$

6. $(.0072)^{12} =$

7. $(19)^{5.4} =$

8. $5^{7.3} =$

9. $(124)^{16.3} =$

10. $(76)^{12.7} =$

11. $(0.15)^{7.5} =$

12. $(145)^{20} =$

13. $(1230)^{-5.2} =$

14. $(1.25)^{51.6} =$

15. $(8.4)^{27} =$

16. $(9.6)^{-15} =$

17. $(.0064)^{8.75} =$

18. $(.062)^{-21.4} =$

19. $(.00053)^{-6.7} =$

20. $(27.5)^{14.6} =$

B.2 Numbers very near 1 (rules with 6 LL scales)

Numbers near 1 may be located on LL1 down to 1.01, and on LLo1 up to 0.99; therefore, any number between 0.99 and 1.01 is outside the scale range. In this section we illustrate procedures for working with such off-scale numbers, and we shall make use of the following approximate relations:

> If x is a positive number near zero:
>
> 1. $(1 + x)^n \approx 1 + nx$
> 2. $(1 - x)^n \approx 1 - nx$

Example 1: $(1.008)^{1.2} = ?$

Use relation (1):

$$(1.008)^{1.2} = (1 + .008)^{1.2} \approx 1 + (1.2)(.008) = \boldsymbol{1.0096} \text{ (approx.)}$$

Example 2: $(0.9996)^{3.5} = ?$

Use relation (2):

$$(0.9996)^{3.5} = (1 - .0004)^{3.5} \approx 1 - (3.5)(.0004) = \boldsymbol{0.9986} \text{ (approx.)}$$

Example 3: $(1.008)^{75} = ?$

Here, the exponent is quite large, and the result is in the range of the LL scales. We break up the exponent in the following way:

$$(1.008)^{75} = [(1.008)^k]^{75/k}$$

Now choose k just large enough to get $(1.008)^k$ on to the *left end* of the LL1 scale. Clearly, $k = 2$ will be satisfactory. Using relation (1) with $k = 2$:

$$(1.008)^2 = (1 + .008)^2 \approx 1 + 2(.008) = 1.016 \text{ (approx.)}$$

Hence, the original power may be written:

$$(1.008)^{75} \approx (1.016)^{75/2} = (1.016)^{37.5} \text{ (approx.)}$$

This may be evaluated on the LL scales in the usual manner.

Verify that the result is ***1.812.***

(This is, of course, an approximation. Using 5-place log tables, the answer is 1.818.)

Example 4: $(1.0045)^{146} = ?$

Write: $(1.0045)^{146} = [(1 + .0045)^k]^{146/k}$.

Now if we take $k = 3$, we will just get on to the left end of LL1. Using relation (1) with $k = 3$:

$$(1 + .0045)^3 \approx 1 + 3(.0045) = 1.0135 \text{ (approx.)}$$

Hence, we may write:

$$(1.0045)^{146} \approx (1.0135)^{146/3}$$

Evaluating as usual on the LL scales, verify that result is ***1.920*** (approx.) (Using 5-place logs, the answer is 1.9262.)

Example 5: $(1.00031)^{850} = ?$

Write: $(1.00031)^{850} = [(1 + .00031)^k]^{850/k}$.

In this case, $k = 40$ will bring us on to the left end of LL1.

Using relation (1): $(1 + .00031)^{40} \approx 1.0124$ (approx.)

Verify that $(1.00031)^{850} \approx (1.0124)^{850/40} =$ ***1.299*** (approx.)

Verify the following:

1. $(1.0005)^{4.4} \approx 1.0022$

2. $(0.9972)^{0.5} \approx 0.9986$

3. $(1.0026)^2 \approx 1.0052$

4. $(1.0063)^{60} \approx 1.455$

5. $(1.0023)^{115} \approx 1.301$

6. $(1.00037)^{700} \approx 1.294$

Example 6: $(0.99944)^{600} = ?$

In this case, we break up the exponent so that we can just get on to the *left end* of LL01.

Write: $(0.99944)^{600} = [(1 - .00056)^k]^{600/k}$.

Here, $k = 20$ will just bring us on to the left end of LL01.

Using relation (2) with $k = 20$:

$$(1 - .00056)^{20} \approx 1 - 20(.00056) = 0.9888 \text{ (approx.)}$$

The original power may now be written:

$$(0.99944)^{600} \approx (0.9888)^{600/20} = (0.9888)^{30} \text{ (approx.)}$$

Evaluating on the LL scales in the usual manner, verify that result is ***0.713*** (approx.)